NEW SYLLABUS MATHEMATICS 3 (6th Edition)
Specific Instructional Objectives (SIOs)

Authors: Teh Keng Seng *BSc,Dip Ed*
Loh Cheng Yee *BSc,Dip Ed*

SET A

This file contains a specified/suggested teaching schedule for the teachers.

OXFORD UNIVERSITY PRESS
No. 38, Sector 15, Korangi Industrial Area
P.O. Box 8214, Karachi 74900
Pakistan
(021) 111 693 673 *uan*
(021) 5071580-86 *telephone*
(021) 5055071-2 *fax*
oup.pk@oup.com *e-mail*

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Term 1 **Week 1, 2 & 3**	**Chapter 1** **Solutions to Quadratic Equations**	• Solve quadratic equations by factorisation (revision). • Form a quadratic equation when the roots are given • Complete a given expression of the form $(x^2 + kx)$ to obtain a perfect square. • Solve a quadratic equation by 'completing the square' method. • Solve a quadratic equation by using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. • Solve a non-quadratic equation by reducing it to a quadratic equation. . • Solve problems involving quadratic equations.	1a 1a 1b 1c 1d 1e 1f	*NE* Pg 14 Discuss why we have COE & ERP and whether they are necessary and effective.		Pg 5, 14	Pg 14 Example 14 Pg 17 Exercise 1f Q16	Pg 3, 10 Refer to TG	Textbook
Term 1 **Week 4 & 6**	**Chapter 2** **Indices and Standard Form**	• Use the Multiplication Law of Indices to simplify terms that involve positive indices. • Use the Division Law of Indices to simplify terms that involve positive indices. • Use the Power Law of Indices to simplify terms that involve positive indices. • Use the various Laws of Indices to simplify terms that involve positive indices. • State the Laws of Indices involving zero and negative indices and use them to evaluate numerical expressions with zero and negative indices.	2a 2b 2c 2c 2d	*NE* Pg 22 Discuss the need for taxes and how the taxes are used in Singapore.	Pg 33, 36, 41-42, 44	Pg 32, 36, 39, 43	Pg 22 Introduction Pg 49 Exercise 2h Q27 Pg 52 Review Questions 2 Q6		Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• State the Law of Indices involving fractional indices and use it to evaluate and simplify expressions involving them. • Solve equations involving indices. • Use the standard form to express very large or very small numbers. • Use the calculator to evaluate numbers involving standard form and power of a number.	2e 2f 2g 2h						
Term 1 **Week 7 & 8**	**Chapter 3** **Linear Inequalities**	• State the properties of inequalities: (1) if $x > y$ and $y > z$, then $x > z$. (2) if $x > y$, then $x + z > y + z$ and $x - z > y - z$, (3) if $x > y$ and $z > 0$, then $xz > yz$ and $\frac{x}{z} > \frac{y}{z}$, (4) if $x > y$ and $z < 0$, then $xz < yz$ and $\frac{x}{z} < \frac{y}{z}$, and use them to solve simple inequalities. • Distinguish the difference between < and $\leq$ and use a number to represent them. • Solve problems involving inequalities. • Solve linear inequalities involving one variable.	3a 3b 3c 3d	Discuss and give examples on how inequalities are used in everyday life situations.	Pg 55, 63	Pg 57, 59, 60, 62	Pg 61 Example 6 Pg 63 Exercise 3c Q1 & Q2 Pg 70 Review Questions 3 Q10		Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Term 1 **Week 9 & 10**	**Chapter 4** **Coordinate Geometry**	• Locate the position of a coordinate point on a graph and find the length of a line segment. • Find the gradient of a line joining two given points. • Find the equation of a straight line given its gradient *m* and one point on the line. • Find the equation of a straight line joining two given points. • Solve related problems involving equations of straight lines.	4a 4b 4c 4c 4c	Ask pupils to cite examples of how the idea of coordinate geometry is used in everyday life situations.	Pg 79, 85			GSP: Pg 83, 84 Refer to TG	Textbook
Term 2 **Week 1 & 2**	**Chapter 5** **Matrices**	• State the properties and characteristics of Row, Column, Square, Equal and Null Matrices. • State the order of a matrix. • Add and subtract two matrices of the same order. • Multiply a matrix by a real number. • Multiply two matrices. • Solve everyday life problems by using matrices.	5a 5a 5b 5c 5d 5e	Discuss how the idea of matrices is being used in spreadsheets and how these programs are useful in our everyday lives.	Pg 109, 110	Pg 95			Textbook
Term 2 **Week 3 & 4**	**Chapter 6** **Application of Mathematics in Practical Situations**	• Solve problems involving profit and loss. • Solve problems involving further examples of percentages. • Solve problems involving simple interest.	6a 6b 6c	Discuss the power of compound interest. Ask pupils to calculate the amount that one has to pay if one	Pg 135, 137, 139-140, 149,	Pg 132, 151	Pg 134 Exercise 6b Q8 & Q9 Pg 147 on taxation		Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Solve problems involving compound interest. • Solve problems involving hire purchase. • Convert one currency to another. • Calculate simple taxation problems. • Solve problems involving personal and household finances. • Interpret and use tables and charts in solving problems. • Use different problem solving strategies to solve everyday life problems.	6d 6e 6f 6g 6h 6i	owes money to the credit card company where interest is charged at 24% per annum and compounded monthly. Ask why many people are made bankrupt in the face of credit card debts.			Pg 153 Exercise 6h Q12 & Q13		
Term 2 **Week 5 & 6**	**Chapter 7** **Linear Graphs and Their Applications**	• Interpret and use conversion graphs. • Interpret and use travel graphs. • Draw graphs to represent practical problems. • Solve problems involving linear graphs such as travel graphs and graphs in practical situations.	7a 7b 7c 7d		Pg 171	Pg 176, 183			Textbook
Term 2 **Week 7 & 8**	**Chapter 8** **Congruent and Similar Triangles**	• Identify congruent triangles. • State and use the congruency tests: SSS, SAS, AAS and RHS to test if two triangles are congruent. • Apply the congruency tests to solve given triangles. • Identify similar triangles. • State the tests for similarity between two triangles.	8a 8a, 8b 8c 8d	Discuss how congruent and similar figures are found and used in everyday life situations.	Pg 204-205, 209-210, 220-221	Pg 206, 216, 217 219, 220, 227			Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Use the rules for similarity between two triangles to solve problems involving similar triangles.	8e						
Term 3 **Week 1, 2 & 3**	**Chapter 9** **Area and Volume of Similar Figures and Solids**	• State that the ratio of the areas of any two similar figures is equal to the square of the ratio of any two corresponding lengths of the figures. • Use the above rule to solve problems involving the area and lengths of two similar figures. • State that the ratio of the volumes of any two similar solids is equal to the cube of the ratio of any two corresponding lengths of the solids. • Use the above rule to solve problems involving the volumes, areas and lengths of two similar solids.	9a 9b		Pg 242, 250	Pg 245, 251 Pg 241, 242, 244, 251			Textbook
Term 3 **Week 4, 5 & 6**	**Chapter 10** **Trigonome-trical Ratios**	• Define the three basic trigonometrical ratios in terms of the lengths of the hypotenuse side, opposite side and adjacent side with respect to an acute angle of a right-angled triangle. • Find the value of a trigonometrical ratio using a calculator. • Find the length of a side of a right-angled triangle using trigonometrical ratios. • Find the value of an angle of a right-angled triangle using trigonometrical ratios. • Solve problems involving angles and lengths of a right-angled triangle. • Solve practical everyday life problems using trigonometrical ratios.	10a 10b 10c 10d 10e 10f		Pg 261, 273, 278	Pg 262, 273 Pg 272		GSP: Pg 265-266	Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Solve more complicated problems with the use of trigonometry.	10g						
Term 3 **Week 7, 8 & 9**	**Chapter 11** **Further Trigonome-try**	• Find the value of trigonometrical ratios of an obtuse angle.	11a		Pg 304, 308-309, 311	Pg 321, 322		GSP: Pg 306-307	Textbook
		• State the formula for finding the area of a triangle: Area of $\Delta ABC = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$ and use it to solve the angles or sides of a triangle.	11b						
		• State the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and use it to solve a triangle given two sides and one non-included angle or one side and two angles.	11c						
		• Identify whether the ambiguous case occurs for a particular triangle and solve a triangle involving the ambiguous case.	11c						
		• State the cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$ and use it to solve a triangle given two sides and an included angle or given three sides.	11d						
		• Find the bearing of one point from another and use the sine and cosine rules to solve problems involving bearing.	11e						
		• Solve simple problems involving 3-D figures in the form of a cube, cuboid, right pyramid, circular cone and cylinder.	11f						
		• Find the angle of elevation and depression in simple 3D problems.	11g						

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Term 3 **Week 10** **&** **Term 4** **Week 1**	**Chapter 12** **Mensuration - Arc Length, Sector Area, Radian Measure**	• Find the area and circumference of a circle, a quadrant and a semi-circle. • Find the arc length and area of a sector. • Define a radian and to convert an angle in radian to degree and vice versa. • Use the formula $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ to solve problems involving arcs and sectors with angles expressed in radians.	12a 12b 12c 12d		Pg 338, 340-341	Pg 341, 343, 352			Textbook
Term 4 **Week 2, 3 & 4**	**Chapter 13** **Geometrical Properties of Circles**	• State the symmetric properties of a circle, (i) a straight line drawn from the centre of a circle to bisect a chord is perpendicular to the chord, (ii) equal chords are equidistant from the centre of a circle or centres of equal circles. • Calculate the perpendicular distance between the centre of a circle and a chord and solve related problems. • State the angle properties of a circle, (i) an angle at the centre of a circle is twice any angle at the circumference subtended by the same arc, (ii) a triangle in a semicircle with the diameter as one of its sides, has a right angle at the circumference, (iii) angles in the same segment of a circle are equal, and use the above properties to solve related problems. • State that angles in opposite segments of a circle are supplementary and use the property to solve problems involving angles of a quadrilateral on a circle and related problems on the property.	13a 13a 13b 13c		Pg 371, 377-378, 382	Pg 373		GSP: Pg 365-367	Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Use all the above properties to prove mathematical statements involving angle properties of circles.	13d						
		• State the property that a tangent to a circle is perpendicular to the radius drawn to the point of contact. State the properties regarding tangents drawn from an external point, (i) tangents drawn to a circle from an external point are equal in length, (ii) tangents subtend equal angles at the centre, (iii) the line joining the external point to the centre of the circle bisects the angle between the tangents, and use the above properties to solve problems involving tangents to a circle.	13e						

NEW SYLLABUS MATHEMATICS 2 & 3 (6th Edition)
Specific Instructional Objectives (SIOs) for Normal (Academic) Level

SET A

This file contains a specified/suggested teaching schedule for the teachers.

OXFORD UNIVERSITY PRESS
No. 38, Sector 15, Korangi Industrial Area
P.O. Box 8214, Karachi 74900
Pakistan
(021) 111 693 673 *uan*
(021) 5071580-86 *telephone*
(021) 5055071-2 *fax*
oup.pk@oup.com *e-mail*

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Term 1 **Week 1, 2 & 3**	**Chapter 6 Of Book 2** **Pythagoras' Theorem**	• Identify a right-angled triangle and its hypotenuse. • Define the Pythagoras' theorem and its converse and use proper symbols to express the relationship. • Apply the Pythagoras' theorem to find the unknown side of a right-angled triangle when the other two sides are given. • Solve word problems involving right-angled triangles using Pythagoras' theorem.	6a 6a 6b 6b		Pg 178 Pg 181: Find out how mathematics and music are related, how computer music are made etc. Pg 185: Find out more about Pythagorean Triples.				Textbook
Term 1 **Week 4, 5 & 6**	**Chapter 9 Of Book 2** **Graphs of Quadratic Functions**	• Identify important features of quadratic graphs $y = ax^2$ when a takes on positive and negative values. • Construct a table of values for x and y for a quadratic function. • Plot a quadratic graph from a table of values with/without the aid of a curved rule. • Identify the equation of a line of symmetry of a quadratic graph. • Find the values of x and y from the quadratic graph by locating the point/s of intersection of a graph and a straight line. • Express word problems into quadratic equation and solve the problem using graphical method.	9a 9a 9a 9b 9b		Pg 261-262, 264-265	Pg 263, 264		Graph-matica: Pg 262	Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Term 1 **Week 7, 8 & 9**	**Chapter 6 Of Book 3** **Application of Mathematics in Practical Situations**	• Solve problems involving profit and loss. • Solve problems involving further examples of percentages. • Solve problems involving simple interest. • Solve problems involving compound interest. • Solve problems involving hire purchase. • Convert from one currency to another. • Calculate simple taxation problems. • Solve problems involving personal and household finances. • Interpret and use tables and charts in solving problems. • Use different problem solving strategies to solve everyday life problems.	6a 6b 6c 6d 6e 6f 6g 6h 6i	Discuss the power of compound interest. Ask pupils to calculate the amount that one has to pay if one owes money to the credit card company where interest is charge at 24% per annum compounded monthly and why many people are made bankrupt in the face of credit card debts. Compare this to the rate that the loan sharks charged.	Pg 135, 137, 139-140, 149	Pg 132, 151	Pg 134 Exercise 6b Q8 & Q9 Pg 147 on taxation Pg 153 Exercise 6h Q12 & Q13		Textbook
Term 1 **Week 10 and** **Term 2**	**Chapter 1 Of Book 3** **Solutions to Quadratic Equations**	• Solve quadratic equations by factorisation (revision). • Form a quadratic equation when the roots are given. • Complete a given expression of the form $(x^2 + kx)$ to obtain a perfect square. • Solve a quadratic equation by 'completing the square'	1a 1b 1c	*NE* pg 14 Discuss why we have COE & ERP and whether they are necessary and effective.		Pg 5, 14	Pg 14 Example 14 Pg 17 Exercise 1f Q16	Pg 3, 10 Refer to TG	Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Week 1 & 2		method. • Solve a quadratic equation by using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. • Solve non-quadratic equations by reducing it to a quadratic equation. • Solve problems involving quadratic equations.	1d 1e 1f						
Term 2 **Week 3 & 4**	**Chapter 2 Of Book 3** **Indices And Standard Form**	• Use the Multiplication Law of Indices to simplify terms that involve positive indices. • Use the Division Law of Indices to simplify terms that involve positive indices. • Use the Power Law of Indices to simplify terms that involve positive indices. • Use the Various Laws of Indices to simplify terms that involve positive indices. • State the Laws of Indices involving zero and negative indices and use them to evaluate numerical expressions with zero and negative indices. • State the Law of Indices involving fractional indices and use it to evaluate and simplify expressions involving them. • Solve equations involving indices. • Use the standard form to express very large or very small numbers.	2a 2b 2c 2c 2d 2e 2f 2g	*NE* Pg 22 Discuss the need for taxes and how the taxes are used in Singapore.	Pg 33, 36, 41-42, 44	Pg 32, 36, 39, 43	Pg 22 Introduction Pg 49 Exercise 2h Q27 Pg 52 Review Questions 2 Q6		Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Use the calculator to evaluate numbers involving standard form and powers of a number.	2h						
Term 2 **Week 5, 6 & 7**	**Chapter 4 Of Book 3** **Coordinate Geometry**	• Locate the position of a coordinate point on a graph and find the length of a line segment. • Find the gradient of a line joining two given points. • Find the equation of a straight line given its gradient *m* and one point on the line. • Find the equation of a straight line joining two given points. • Solve related problems involving equations of straight lines.	4a 4b 4c 4c 4c	Ask pupils to cite examples of how the idea of coordinate geometry is used in everyday life situations.	Pg 79, 85			GSP: Pg 83, 84 Refer to TG	Textbook
Term 3 **Week 1, 2 & 3**	**Chapter 7 Of Book 3** **Linear Graph and their Applications**	• Interpret and use conversion graphs. • Interpret and use travel graphs. • Draw graphs to represent to represent practical problems • Solve problems involving linear graphs such as travel graphs and graphs in practical situations.	7a 7b 7c 7d		Pg 171	Pg 176, 183			Textbook
Term 3 **Week 4, 5 & 6**	**Chapter 9 Of Book 3** **Area and Volume of similar figures and solids**	• State that the ratio of the area of any two similar figures is equal to the square of the ratio of any two corresponding lengths of the figures. • Use the above rule to solve problems involving the area and lengths of two similar figures.	9a		Pg 242, 250	Pg 245, 251 Pg 241, 242, 244, 251			Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• State that the ratio of the volumes of any two similar solids is equal to the cube of the ratio of any two corresponding lengths of the solids. • Use the above rule to solve problems involving the volumes, areas and lengths of two similar solids.	9b						
Term 3 **Week 7, 8, 9 & 10**	**Chapter 10 Of Book 3** **Trigono-metrical Ratios**	• Define the three basic trigonometrical rations in terms of hypotenuse side, opposite side and adjacent side with respect to an acute angle of a right-angled triangle. • Find the value of a trigonometrical ratio using a calculator. • Find the length of a side of a right-angled triangle using trigonometrical ratios. • Find the value of an angle of a right-angled triangle using trigonometrical ratios. • Solve problems involving angles and lengths of a right-angled triangle. • Solve practical everyday life problems using trigonometrical ratios. • Solve more complicated problems with the use of trigonometry.	10a 10b 10c 10d 10e 10f 10g		Pg 261, 273, 278	Pg 262, 273 Pg 272		GSP: Pg 265-266	Textbook
Term 4 **Week 1, 2, 3**	**Chapter 13 Of Book 3** **Geometrical Properties of**	• State the symmetric properties of a circle, (i) a straight line drawn from the centre of a circle to bisect a chord is perpendicular to the chord, (ii) equal chords are equidistant from the centre of a circle or centres of equal circles.	13a		Pg 371, 377-378, 382	Pg 373		GSP: Pg 365-367	Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
& 4	**Circles**	• Calculate the perpendicular distance between the centre of a circle and a chord and solve related problems.	13a						
		• State the angle properties of a circle, (i) an angle at the centre of a circle is twice any angle at at circumference subtended by the same arc, (ii) every angle in a semicircle is a right angle, (iii) angles in the same segment of a circle are equal, and use the above properties to solve related problems.	13b						
		• State the properties of angles in opposite segments of a circle are supplementary and use the above property to solve problems involving angles of a quadrilateral on a circle and related problems on angle properties of circles.	13c						
		• Use all the above properties to prove mathematical statements involving angle properties of circles.	13d						
		• State the property that a tangent to a circle is perpendicular to the radius drawn to the point of contact.							
		• State the properties regarding tangents drawn from an external point, (i) tangents drawn to a circle from an external point are equal in length, (ii) tangents subtend equal angles at the centre, (iii) the line joining the external point to the centre of the circle bisects the angle between the tangents, and use the above properties to solve problems involving tangents to a circle.	13e						

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
For Sec 4N(A)	**Chapter 11 Of Book 3** **Further Trigonome-try**	• Find the value of trigonometrical ratios of an obtuse angle.	11a		Pg 304, 308-309, 311	Pg 321, 322		GSP: Pg 306-307	Textbook
		• State the formula for finding the area of a triangle $\Delta ABC = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$ and use it to solve a triangle.	11b						
		• State the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and use it to solve a triangle given two sides and non-included angle or one side and two angles.	11c						
		• Identify whether the ambiguous case occurs for a particular triangle and solve a triangle involving ambiguous case.	11c						
		• State the cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$ and use it to solve a triangle given two sides and an included angle or given three sides.	11d						
		• Find the bearing of one point from another and use the sine and cosine rules to solve problems involving bearing.	11e						
		• Solve simple problems involving 3-D figures in the form of a cube, cuboid, right pyramids, circular cones and cylinders.	11f						
		• Find the angle of elevation and depression in simple 3D problems.	11g						
For Sec 4N(A	**Chapter 12 Of Book 3** **Mensuration-Arc Length,**	• Find the area and circumference of a circle, a quadrant and a semi-circle.	12a		Pg 338, 340-341	Pg 341, 343, 352			Textbook
		• Find the length and area of a sector.	12b						

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
	Sector Area, Radian Measure	• Define a radian and to convert an angle in radian to degree and vice versa. • Use the formula $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ to solve problems involving arcs and sectors expressed in radians.	12c 12d						
For Sec 5N(A	**Chapter 3 Of Book 3** **Linear Inequalities**	• State the properties of inequalities: (1) if $x > y$ and $y > z$, then $x > z$. (2) if $x > y$, then $x + z > y + z$ and $x - z > y - z$, (3) if $x > y$ and $z > 0$, then $xz > yz$ and $\frac{x}{z} > \frac{y}{z}$, (4) if $x > y$ and $z < 0$, then $xz < yz$ and $\frac{x}{z} < \frac{y}{z}$, and use them to solve simple inequalities. • Distinguish the difference between < and ≤ and use a number to represent them. • Solve problems involving Inequalities. • Solve linear inequalities involving one variable.	3a 3b 3c 3d	Discuss and give examples on how inequalities are used in everyday life situation.	Pg 55, 63,	Pg 57, 59, 60, 62	Pg 61 Example 6 Pg 63 Exercise 3c Q1 & Q2 Pg 70 Review Questions 3 Q10		Textbook
For Sec 5N(A	**Chapter 5 Of Book 3** **Matrices**	• State the properties and characteristics of Row, Column, Square, Equal and Null Matrices. • State the order of a matrix. • Add and subtract two matrices of the same order. • Multiply a matrix by a real number.	5a 5a 5b 5c	Discuss how the idea of matrices is being use in spread sheets and how these programmes are so useful in our everyday lives.	Pg 109, 110	Pg 95			Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Multiply two matrices. • Solve everyday life problems by using matrices.	5d 5e						
For Sec 5N(A	**Chapter 8 Of Book 3** **Congruent and Similar Triangles**	• Identify congruent triangles. • State and use the congruency tests: SSS, SAS, AAS and RHS to test if two triangles are congruent. • Apply the congruency tests to solve given triangles. • Identify similar triangles. • State the tests for similarity between two triangles. • Use the rules for similarity between two triangles to solve problems involving similar triangles.	8a 8a 8b 8c 8d 8e	Discuss how congruent and similar figures are found and use in everyday life situations.	Pg 204-205, 209-210, 220-221	Pg 206, 216, 217 219, 227			Textbook

NEW SYLLABUS MATHEMATICS 3 & 4 (6th Edition)
Specific Instructional Objectives (SIOs) for Normal (Academic) Level

SET A

This file contains a specified/suggested teaching schedule for the teachers.

OXFORD UNIVERSITY PRESS
No. 38, Sector 15, Korangi Industrial Area
P.O. Box 8214, Karachi 74900
Pakistan
(021) 111 693 673 *uan*
(021) 5071580-86 *telephone*
(021) 5055071-2 *fax*
oup.pk@oup.com *e-mail*

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Term 1 **Week 1 & 2**	**Part of Chapter 4 Of Book 2** **Algebraic Manipulation and Formulae**	• Change the subject of a simple formula. • Changing the subject of a formula involving squares, square roots, cubes and cube roots etc. • Finding the unknown in a formula.	4i 4j 4k			Pg 138, 139, 143			Textbook
Term 1 **Week 3, 4, 5 & 6**	**Chapter 1 Of Book 4** **Graphical Solution of Equations**	• Construct a table of values of x and y for (i) a cubic function, $y = ax^3 + bx^2 + cx + d$, (ii) a reciprocal function, $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$, (iii) an exponential function, $y = a^x$, and plot the graphs of these functions on a piece of graph paper. • Find the value(s) of x for a given value of y and the value of y for a given value of x from the graphs above. • Sketch graphs of quadratic functions of the form $y = ax^2$, $y = \pm(x-a)(x-b)$ and $y = \pm(x-p)^2 + q$ where a, b, p and q are constants. • Draw the graphs of a quadratic function and use it to solve related quadratic equations graphically. • Draw the graphs of cubic, reciprocal and exponential functions and use them to solve related equations graphically.	1a 1a 1b 1b 1b	Where do you find uses of graphs in everyday life situations?	Use Graphmatica to see the shape of graphs and to solve equations graphically.	Pg 26, 31 Pg 17, 37		Graph-matica : Pg 3, 6, 9, 10, 19, 22, 25, 28	Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Term 1 **Week 7 & 8**	**Chapter 2 Of Book 4** **Further Graphs and Graphs Applied to Kinematics**	• Convert speeds from km/h to m/s and vice versa. • Find the gradients of a curve by drawing a tangent to the curve. • Draw the distance-time graph from given information and use it to find the velocity and solve related problems. • Interpret a velocity-time graph and use it to find the distance moved by calculating the area under the curve; find the instantaneous acceleration at any point of time by finding the gradient of the tangent of the velocity-time graph at that time. • Draw a velocity-time graph from given information and use it to solve problems on distance, average speed and acceleration. • Solve problems relating to graphs in practical situations.	2a 2a 2a 2b 2b 2b	Pg 43, 45 **Just For Fun** Ask for various answers and let pupils explain how they got them.		Pg 42, 58, 68, 70	Pg 50 Exercise 2a Q3 & Q5 Pg 68 Review Questions 2 Q4	Graph-matica : Pg 47-48	Textbook
Term 1 **Week 9 & 10** **&** **Term 2** **Week 1 & 2**	**Chapter 11 Of Book 3** **Further Trigonome-try**	• Find the value of trigonometrical ratios of an obtuse angle. • State the formula for finding the area of a triangle $\Delta ABC = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$ and use it to solve a triangle. • State the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and use it to solve a triangle given two sides and non-included angle or one side and two angles.	11a 11b 11c		Pg 304, 308-309, 311	Pg 321, 322		GSP: Pg 306-307	Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Identify whether the ambiguous case occurs for a particular triangle and solve a triangle involving ambiguous case.	11c						
		• State the cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$ and use it to solve a triangle given two sides and an included angle or given three sides.	11d						
		• Find the bearing of one point from another and use the sine and cosine rules to solve problems involving bearing.	11e						
		• Solve simple problems involving 3-D figures in the form of a cube, cuboid, right pyramids, circular cones and cylinders.	11f						
		• Find the angle of elevation and depression in simple 3D problems.	11g						
Term 2	**Chapter 12 Of Book 3**	• Find the area and circumference of a circle, a quadrant and a semi-circle.	12a		Pg 338, 340-341	Pg 341, 343, 352			Textbook
Week 3 & 4	**Mensuration-Arc Length, Sector Area, Radian Measure**	• Find the length and area of a sector.	12b						
		• Define a radian and to convert an angle in radian to degree and vice versa.	12c						
		• Use the formula $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ to solve problems involving arcs and sectors expressed in radians.	12d						

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Term 2 **Week 5, 6 & 7**	**Chapter 5 Of Book 4** **Cumulative Frequency Distribution**	• Construct a cumulative frequency table from a given frequency distribution table. • Draw a cumulative frequency curve and use it to estimate the number or percentage of particular participants exceeding or falling short of a figure. • Find the median, lower and upper quartiles and percentiles from a cumulative frequency curve and use them to find inter-quartile range and solve other related problems. • Able to comment and compare the performance of two sets of data based on the median and inter-quartile range of the data. • Draw a box-and-whisker plots from a set of data. • Able to comment and compare the performance of two sets of data based on box-and-whisker plots of the sets of data.	5a 5a 5b 5b 5c 5c		Pg 174	Pg 189, 206		Excel: Pg 182-183	Textbook
Term 3 **Week 1, 2 & 3**	**Chapter 6 Of Book 4** **More on Probability**	• Define the classical definition of probability of an event E occurring as $P(E)=\frac{\text{No. of outcomes favouable to the occurence of E}}{\text{Total number of equally likely outcomes}}$ • List the elements in the sample space of an experiment. • Use the possibility diagrams to list the sample space of simple combined events.	6a 6b 6c	Discuss "Is it worthwhile to gamble? What are the odds? Is it better to bet on 4-digit 'BIG' or 'SMALL'?" Refer to Pg 362 and TG.	Pg 224, 231, 232,	Pg 221, 234-235,			Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Use the tree diagrams to list the sample space of simple combined events. • Perform calculation using the addition law to find the probability of mutually exclusive events. • Perform calculation using the multiplication law to find the probability of independent events. • State that for any event E, $0 \leq P(E) \leq 1$. • P(E)=0 if and only if the event E cannot possibly occur. • P(E)=1 if and only if the event E will certainly occur. • State the rule P(E) = 1 – P(E') where E and E' are complementary events. • Use all the above theory to solve problems involving two or more events.	6d 6d 6e						
Term 3 **Week 4 to 10**	**Chapter 7 Of Book 4** **Revision**		7a onwards	Pg 349: Should we be proud of ourselves for being great gamblers? Pg 353: Can you give concrete examples where statistics are being distorted?			Pg 306, 317 322, 326, 327		Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
For Sec 5N(A)	**Chapter 3 Of Book 4** **Vectors in Two Dimensions**	• Differentiate between scalars and vectors and give two examples of each. • Represent a vector using proper terminologies and notations. • Define and identify equal vectors. • Define and identify negative of a vector and the zero vector. • Express a vector in column vector form. • Find the magnitude and direction of a vector in column vector form. • Use triangle law of vector addition to find the sum of and difference between two vectors. • Multiply a column vector by a scalar. • Express a given vector in terms of two component vectors. • Define position vector. • Find the resultant of two position vectors.	3a 3a 3a 3a 3b 3c 3d 3e 3e	Pg 75	Pg 103	Pg 79			Textbook
For Sec 5N(A)	**Chapter 4 Of Book 4** **Standard Deviation and Mean**	• Find the mean of a given grouped data. • Calculate the standard deviation of a set of data. • Calculate the standard deviation of a set of grouped data.	4a 4b 4b	Discuss how some statistics may be manipulated or misrepresented. What are the properties of				Excel: Pg 144	Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Able to comment and compare the performance of two sets of data based on the mean and standard deviation.	4b	standard deviation and how they are used in everyday situations.					

NEW SYLLABUS MATHEMATICS 2, 3 & 4 (6th Edition)
Specific Instructional Objectives (SIOs) for Normal (Academic) Level

SET A

This file contains a specified/suggested teaching schedule for the teachers.

OXFORD UNIVERSITY PRESS
No. 38, Sector 15, Korangi Industrial Area
P.O. Box 8214, Karachi 74900
Pakistan
(021) 111 693 673 *uan*
(021) 5071580-86 *telephone*
(021) 5055071-2 *fax*
oup.pk@oup.com *e-mail*

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Term 1 **Week 1, 2 & 3**	**Chapter 8 Of Book 3** **Congruent And Similar Triangles**	• Identify congruent triangles. • State and use the congruency tests: SSS, SAS, AAS and RHS to test if two triangles are congruent. • Apply the congruency tests to solve given triangles. • Identify similar triangles. • State the tests for similarity between two triangles. • Use the rules for similarity between two triangles to solve problems involving similar triangles.	8a 8a 8b 8c 8d 8e	Discuss how congruent and similar figures are found and use in everyday life situations.	Pg 204-205, 209-210, 220-221	Pg 206, 216, 217 219, 227			Textbook
Term 1 **Week 4, 5 & 6**	**Chapter 10 Of Book 2** **Set Language and Notation**	• Define the term ‘set’. • Write a statement using proper set notations and symbols. • Use Venn diagrams to represent a set. • Define and identify an empty set and universal set. • Define and identify equal sets, disjoint set and complement of a set and to give examples of these sets. • Define and distinguish subsets and proper subsets of a given set. • Define the intersection and union of sets and the relationships between sets by using Venn diagrams.	10a 10a 10b 10c 10d	The origin and use of sets.	Pg 290	Pg 296, 302	Pg 290 Activity B		Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Use Venn diagrams to solve problems involving classification and cataloguing.							
Term 1 **Week 7, 8 & 9**	**Chapter 3 Of Book 3** **Linear Inequalities**	• State the properties of inequalities: (1) if $x > y$ and $y > z$, then $x > z$. (2) if $x > y$, then $x + z > y + z$ and $x - z > y - z$, (3) if $x > y$ and $z > 0$, then $xz > yz$ and $\frac{x}{z} > \frac{y}{z}$, (4) if $x > y$ and $z < 0$, then $xz < yz$ and $\frac{x}{z} < \frac{y}{z}$, and use them to solve simple inequalities.	3a	Discuss and give examples on how inequalities are used in everyday life situation.	Pg 55, 63,	Pg 57, 59, 60, 62	Pg 61 Example 6 Pg 63 Exercise 3c Q1 & Q2 Pg 70 Review Questions 3 Q10		Textbook
		• Distinguish the difference between < and ≤ and use a number to represent them.	3b						
		• Solve problems involving Inequalities.	3c						
		• Solve linear inequalities involving one variable.	3d						
Term 1 **Week 10** **&** **Term 2** **Week 1 & 2**	**Chapter 5 Of Book 3** **Matrices**	• State the properties and characteristics of Row, Column, Square, Equal and Null Matrices.	5a	Discuss how the idea of matrices is being use in spread sheets and how these programmes are so useful in our everyday lives.	Pg 109, 110	Pg 95			Textbook
		• State the order of a matrix.	5a						
		• Add and subtract two matrices of the same order.	5b						
		• Multiply a matrix by a real number.	5c						
		• Multiply two matrices.	5d						
		• Solve everyday life problems by using matrices.	5e						

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Term 2 **Week 3, 4 & 5**	**Chapter 4 Of Book 4** **Standard Deviation and Mean**	• Find the mean of a given grouped data. • Calculate the standard deviation of a set of data. • Calculate the standard deviation of a set of grouped data. • Able to comment and compare the performance of two sets of data based on the mean and standard deviation.	4a 4b 4b	Discuss how some statistics may be manipulated or misrepresented. What are the properties of standard deviation and how they are used in everyday situations.				Excel: Pg 144	Textbook
Term 3 **Week 1, 2, 3 & 4**	**Chapter 3 Of Book 4** **Vectors in Two Dimensions**	• Differentiate between scalars and vectors and give two examples of each. • Represent a vector using proper terminologies and notations. • Define and identify equal vectors. • Define and identify negative of a vector and the zero vector. • Express a vector in column vector form. • Find the magnitude and direction of a vector in column vector form. • Use triangle law of vector addition to find the sum of and difference between two vectors. • Multiply a column vector by a scalar. • Express a given vector in terms of two component vectors.	3a 3a 3a 3a 3b 3c 3d 3e	Pg 75	Pg 103	Pg 79			Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Define position vector. • Find the resultant of two position vectors.	3e						
Term 3 **Week 4 to 10**	**Chapter 7 Of Book 4** **Revision**		7a onwards	Pg 349 : Should we be proud of ourselves for being great gamblers? Pg 353 : Can you give concrete examples where statistics are being distorted?			Pg 306, 317 322, 326, 327		Textbook

Chapter 1

Secondary 3 Mathematics
Chapter 1 Solutions to Quadratic Equations

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 5)

Take A and B across, time taken = 2 minutes
Take A back, time taken = 1 minute
Take C and D across, time taken = 10 minutes
Take B back, time taken = 2 minutes
Take A and B across, time taken = 2 minutes
Total time taken = 2 + 1 + 10 + 2 + 2 = 17 minutes

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Class: _____ Time allowed: min

Secondary 3 IT Worksheet
Chapter 1 Solutions to Quadratic Equations
Textbook Page 3

Thinking skills used: Inferring, Comparing and Contrasting.

Step 1 Open Graphmatica from the icon on the screen OR from Start, then Program, then Graphmatica

Step 2 Go to **"View", "Graph Paper"** to select **"Rectangular"**, go to **"View"** again to select **"Grid Range".** Select range from –5 to 5 for left and right and from 8 to -12 for top and bottom. You can change these later on your own to see the different effects.

Step 3
- For the curve $y = 2x^2 - 7x$, type y=2x^2-7x and press Enter to see the graph.

Write down the coordinates of the point where the graph cuts the *x*-axis.
(___ , ___), (___ , ___)

- What is the approximate value of *y* when *x* = 3.5? _____
  (You can do this by selecting "**coord curso**" from the tool bar and move the cursor to the point on the graph where *x* = 3.5).
- Sketch the graph in the space below.

The solutions of the equation $2x^2 - 7x = 0$ are *x* = _____ or _____.

Step 4
- Type y=3*x*^2-5*x*-8 for the curve $y = 3x^2 - 5x - 8$ and press Enter to see the graph.
  Write down the coordinates of the point where the graph cuts the *x*-axis.
  (___ ,___), (___ , ___)
- Sketch the graph in the space below.

The solutions of the equation $3x^2 - 5x - 8 = 0$ are *x* = _____ or _____.

Step 5
- Type y=2*x*^2-5*x*-3 and press Enter to see the graph.
  Write down the coordinates of the point where the graph cuts the *x*-axis.
  (___ ,___), (___ , ___)
- Sketch the graph in the space below.

The solution of the equation $2x^2 - 5x - 3 = 0$ are *x* = _____ or _____.

We can also solve the equation $(2x - 1)(x - 2) = 5$ by finding the points of intersection of the curve $y = (2x - 1)(x - 2)$ and $y = 5$. The x-coordinates of the points of intersection of these two graphs will give the solutions of the equation.

Step 6
- Type $y=(2x-1)(x-2)$ and press Enter to see the graph.
- Type $y=5$ and press Enter to see the graph.
  Write down the coordinates of the points where the two graphs intersect.
  (___, ___), (___, ___)
- The solution of the equation $(2x - 1)(x - 2) = 5$ are $x=$ _____ or _____.

You can change the colour of the grid line or the x and y-axes by selecting "**View",** "**colors**" and selecting the desired colours for your graphs, gridlines and background etc.

Conclusion: We can find the solution of the equation $3x^2 – 5x – 8 = 0$ by drawing the graph of $y = 3x^2 – 5x – 8$ and finding the points of intersection of the graph and the line $y = 0$, i.e. the x-axis.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Class: _____ Time allowed: min

Secondary 3 IT Worksheet
Chapter 1 Solutions to Quadratic Equations
Textbook Page 10

Thinking skills used: Inferring, Comparing and Contrasting.

Step 1 Open Graphmatica from the icon on the screen OR from Start, then Program, then Graphmatica

Step 2 Go to **"View", "Graph Paper"** to select **"Rectangular"**, go to **"View"** again to select **"Grid Range".** Select range from –6 to 6 for left and right and from 20 to -20 for top and bottom. You can change these later on to see the different effects.

Step 3
- For the curve $y = 3x^2 - 5x - 7$, we have $a = 3, b = -5, c = -7$.
  What is the value of $(b^2 - 4ac)$? _____
- Type y=3x^2-5x-7 and press Enter to see the graph.
  How many points does the graph cut the x-axis?_____
- What is the approximate value of y when x=3.5? _____
  [You can do this by selecting **"coord curso"** from the tool bar and move the cursor to the point on the graph where x = 3.5. The bottom of the screen shows the co-ordinates where the cursor is placed. Clicking the mouse one more time will let go of this function.]
- Sketch the graph in the space below.

Step 4
- For the curve $y = 9x^2 - 12x + 4$, we have $a = 9, b = -12, c = 4$.
  What is the value of $b^2 - 4ac$? _____
- Type y=9x^2-12x+4 and press Enter to see the graph.
  How many points does the graph cut the x-axis?_____
- What is the approximate value of y when x = 2? _____ (Refer to Step 3 above).
- Sketch the graph in the space below.

Step 5
- For the curve $y = 2x^2 - 8x + 9$, we have $a = 2, b = -8, c = 9$.
  What is the value of $b^2 - 4ac$? _____
- Type y=2x^2-8x+9 and press Enter to see the graph.
  How many points does the graph cut the x-axis?_____
- What is the approximate value of y when x=0.5? _____
- Sketch the graph in the space below.

You can change the colour of the grid line or the *x* and *y*-axes by selecting "**View",** "**colors**" and selecting the desired colours for your graphs, gridlines and background etc. Before you do the next few graphs, clear the screen by selecting "**Clear**" from the tool bar.

Step 6
- For the curve $y = -4x^2 + 13x - 2$, we have $a = -4, b = 13, c = -2$.
  What is the value of $b^2 - 4ac$? _____
- Type *y*=-4*x*^2+13*x*-2 and press Enter to see the graph.
  How many points does the graph cut the *x*-axis?_____
- What is the approximate value of *y* when *x* = 2.5? _____
- Sketch the graph in the space below.

Step 7
- For the curve $y = -4x^2 - 20x - 25$, we have $a = -4, b = -20, c = -25$.
  What is the value of $b^2 - 4ac$? _____
- Type *y*=-4*x*^2-20*x*-25 and press Enter to see the graph.
  How many points does the graph cut the *x*-axis?_____
- What is the approximate value of *y* when *x* = -0.8? _____
- Sketch the graph in the space below

Step 8
- For the curve $y = -6x^2 + 11x - 8$, we have $a = -6, b = 11, c = -8$.
  What is the value of $b^2 - 4ac$? _____
- Type *y*=-6*x*^2+11*x*-8 and press Enter to see the graph.
  How many points does the graph cut the *x*-axis?_____
- What is the approximate value of *y* when *x* = 2.4? _____
- Sketch the graph in the space below.

You may explore more about the shapes of other quadratic graphs by keying in more of such equations on your own.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Class: _____ Time allowed: min

Secondary 3 IT Worksheet
Chapter 1 Solutions to Quadratic Equations

Similarity Between

$y = 3x^2 + 5x - 1$	$y = -3x^2 + 2x + 3$

Differences Between

$y = 3x^2 + 5x - 1$	$y = -3x^2 + 2x + 3$

Conclusion: _____

_____

_____

Similarity Between

$y = x^2 + 3x + 7$	$y = -x^2 + 7x - 13$

Differences Between

$y = x^2 + 3x + 7$	$y = -x^2 + 7x - 13$

Conclusion: _____

_____

_____

Secondary 3 Mathematics
Chapter 1 Solutions to Quadratic Equations

GENERAL NOTES

Teachers should revise the method of solving quadratic equations by factorisation with the pupils. Although solving quadratic equations by 'completing the square method' will not be examined in the GCE 'O' level examinations, its procedure will greatly help pupils to understand the concept of the derivation of the formula. It will also help pupils doing Additional Mathematics understand the topic on quadratic functions better. To stress the importance of 'completing the square' method, the teacher may wish to set a question on it in the class test.

To help pupils memorise the formula for solving quadratic equations, the teacher may wish to ask pupils to write down the formula for every question that they are doing for exercise 1d and 1e.

The teacher may find it useful to use the CD-ROM on quadratic equations produced by CDIS. It will be a good and stimulating IT lesson, as the contents are relevant to our syllabus and it is tailored for local use.

To promote creative thinking, the teacher may ask pupils to set word problems that will lead to quadratic equations, pair off pupils to solve these equations in class and get them to point out any flaws or errors in the questions set.

Common Errors

It is very common for pupils to assign wrong values for a, b, and c in quadratic equations. Emphasise that the general form of a quadratic equation is $ax^2 + bx + c = 0$. For instance, in the equation $x^2 - 3x - 5 = 0$, $a = 1$ (not 0), $b = -3$ (not 3) and $c = -5$ (not 5). For the equation $5x - 3x^2 - 7 = 0$, $a = -3$ (not 5), $b = 5$ (not 3 or –3) and $c = -7$.

At the end of the chapter, the teacher could point out to the pupils that the easiest method to solve a quadratic equation is by factorisation if the equation can be factorised easily. Otherwise, the use of formula is the choice. 'Completing the square' method is only used when a question specifically asks for its use.

After learning the formula, some pupils will just memorise its use and equations which can be solved by easier methods are not noticed. For example, Question 14 of Exercise 1d will lead to $32x^2 + 18 = 26$ which can be solved easily when expressed as $x^2 = \frac{1}{4}$ and $x = \pm\frac{1}{2}$. But some pupils may use the formula to solve this with $a = 32$, $b = 0$ and $c = -8$.

NE MESSAGES

No one owes Singapore a living. We must find our own way to survive.

Page 14 Example 14

Singapore is the first country in the world to introduce the COE and ERP systems as tools to control the vehicle growth in the 1990s. It has been a love-hate system for motorists in Singapore. On the one hand, the motorists love the COE system as it has been effective in curbing the growth of vehicle population and thus keeping the road less congested. On the other hand, the motorists hate the system because the COE is more expensive than the price of the vehicle itself.

Teachers can lead pupils to debate the pros and cons of the COE and ERP system. Can Singapore run as efficiently without the COE and ERP? Are the COE and ERP systems designed by the government to generate more revenue? Ask for any suggestions to improve the system of controlling vehicle growth and at the same time satisfying the desire of Singaporeans to own cars. Would building more roads be a way out?

Page 17 Exercise 1f Q16

We must do our best to preserve the good relations we have with our neighbouring country. We must not speak ill of our neighbour but we must also defend any wrong and unfounded accusations hailed against us. We wish our neighbours well as we are interdependent. Many Singaporeans have relatives in Malaysia. Singapore depends on Malaysia for a great part of her water supply and many Singaporeans have heavy investments in Malaysia. The trade with Malaysia is important for both countries. Singapore leaders and the Sultan of Johor have a long tradition of inviting each other to attend Hari Raya feast in Johor and the Chinese New Year celebration in Singapore.

You may want to discuss with your pupils the issue of the constant traffic jams at the Causeway, the relatively under-used Second Link and the proposed bridge to replace the Causeway.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: 35 min

Class: _____ Marks: /8

Secondary 3 Multiple-Choice Questions
Chapter 1 Solutions to Quadratic Equations

1. The roots of the equation $x^2 - 11x + 30 = 0$ are
   (A) 5, –6 (B) 6, –5 (C) 5, 6 (D) –5, –6 (E) no real roots. ( )

2. What must be added to $3x^2 - 6xa$ to make it a perfect square?
   (A) a^2 (B) $3a^2$ (C) $6a^2$ (D) $12a^2$ (E) 6 ( )

3. Solve the equation $5x^2 - 2x + 1 = 0$, giving your answer correct to 2 decimal places where possible.
   (A) 0.69 or –0.29 (B) 0.29 or –0.69 (C) 0.69 or 0.29
   (D) –0.69 or –0.29 (E) no real roots ( )

4. Solve the equation $3x^2 - 3x - 5 = 0$, giving your answer correct to 2 decimal places where possible.
   (A) 1.88 or 0.88 (B) –1.88 or 0.88 (C) –1.88 or –0.88
   (D) 1.88 or –0.88 (E) no real roots ( )

5. Solve the equation $7 - 5x - 6x^2 = 0$, giving your answer correct to 2 decimal places where possible.
   (A) 0.74 or 1.57 (B) 0.74 or –1.57 (C) –1.57 or –0.74
   (D) 1.57 or –0.74 (E) no real roots ( )

6. The roots of the equation $2 + 6x - x^2 = 0$ are
   (A) $-3 + \sqrt{11}$ (B) $-3 \pm \sqrt{11}$ (C) $\pm 3 + \sqrt{11}$
   (D) $\pm 3 - \sqrt{11}$ (E) $3 \pm \sqrt{11}$ ( )

7. Two pipes P and Q fill a pool at a constant rate of 60 litres per minute and 40 litres per minute respectively. The pool can be filled in 50 minutes, 75 minutes or 30 minutes, depending on whether pipe P alone, pipe Q alone or both pipes P and Q are used. If the pool is filled using pipe P alone for $\frac{1}{3}$ of the time and, both pipes for the rest of the time, how many minutes does it take to fill the pool?
   (A) 30 min (B) $37\frac{1}{2}$ min (C) 35 min
   (D) 40 min (E) none of the above ( )

8. Thomas, John and Larry each drives 150 km of a 450 km journey from Singapore to Kuala Lumpur at speeds of 80, 100 and 120 km/h respectively. What fraction of the total time does Thomas drive?

(A) $\frac{15}{74}$ (B) $\frac{4}{15}$ (C) $\frac{15}{37}$

(D) $\frac{3}{5}$ (E) $\frac{5}{4}$ ( )

Answers

1.	C	2.	B	3.	E	4.	D
5.	B	6.	E	7.	E	8.	C

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: 

Secondary 3 Mathematics Test
Chapter 1 Solutions to Quadratic Equations

1. Factorise completely.
(a) $ab - a - b + 1$ [2]
(b) $b\,(b + 1) - c\,(c + 1)$ [2]
(c) $54 - 6y^2$ [2]

2. Factorise each of the following completely.
(a) $x^3y - 4xy^3$ [2]
(b) $y^2 - x^2 + 6x - 9$ [2]

3. **(a)** Factorise completely $4a^2 - b^2$. [1]
(b) Factorise $3x^2 - 2x - 1$. [2]
(c) Simplify $\frac{9v - 21}{9v^2 - 49}$ [2]

4. Solve the following equations
(a) $(x + 2)\,(x - 2) = 5$ [2]
(b) $(4x + 1)^2 = 9$ [2]
(c) $4x^2 + 4x - 63 = 0$ [2]

5. Solve the equations
(a) $4x^2 - 9x = 0$ **(b)** $\frac{8x - 22 - x^2}{2x - 14}$ [4]

6. Solve the following equations where possible.
(a) $3x^2 + 4x = 8$ [3]
(b) $8x - 3 = x^2$ [3]
(c) $2x^2 - 3x + 5 = 0$ [3]

7. Factorise each of the following completely.
(a) $(x - y + 3)\,(x - y) - 4$ [2]
(b) $4x^2 + 8x$ [1]
(c) $6x^2 + 7x - 5$ [2]
(d) $3a^3 - 12ab^2$ [2]

8. Factorise the following expressions completely.
(a) $2x^2 - 8x$ [1]
(b) $2x^2 + xy - 3y^2$ [2]
(c) $x^2 - 2xy - 35y^2$ [2]

9. Solve the following equations, giving your answers correct to 2 decimal places where necessary.

(a) $x^2 - 10x = 24$ [2]

(b) $3x^2 - 2x = 7$ [3]

(c) $(x + 2)(x + 3) = x + 11$ [2]

10. (a) Simplify $\frac{x^2 - 5x + 6}{(x - 2)(3x + 4)}$ [2]

(b) Solve the equations

(i) $x^2 = 3x$ [2]

(ii) $3x^2 + 13x = 10$ [2]

(iii) $x^2 - x = 6$ [2]

11. Solve the following equations, giving your answers correct to 3 significant figures where necessary.

(a) $12x^2 - x = 20$ [2]

(b) $2x^2 - 7x = 7(5 - x)$ [2]

(c) $5x^2 - 4x = 3(x - 7x^2)$ [2]

12. (a) Factorise $2x^3 - 32x$ completely. [2]

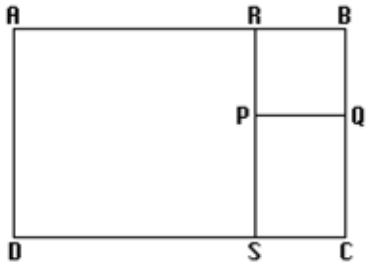
(b) Factorise $x^2 - 12x + 35$. Hence or otherwise, solve the equation $x^4 - 12x^2 + 35 = 0$, giving your answers correct to 2 decimal places. [6]

13. Given that $4x^2 + 12x + k$ is a perfect square, find the value of k. [2]

14. (a) Given that $x + 3y = 5$ and $x - 3y = 2$, find the value of $2x^2 - 18y^2$. [2]

(b) Factorise $4a^2 - (3b - c)^2$ completely. [2]

15. In the diagram, $ABCD$ is a rectangle in which $AB = x$ cm and $BC = 8$ cm. $ARSD$ and $PQBR$ are squares, and the area of $PQCS$ is 15 cm^2. Find the length of PQ in terms of x, and form a quadratic equation in x. Solve this equation to find the possible values of the length AB. [6]



16. Given that $x + y = 6$ and $x^2 - y^2 = 20$, find the value of $4x - 4y$. [2]

17. Solve the equation $t^2 - 7t - 3 = 0$, giving your answers correct to 2 decimal places. [3]

18. Given that $x + y = 8$ and $x^2 - y^2 = 20$, find the value of $3x - 3y$. [3]

19. Solve the equation $2x^2 + 9x - 17 = 0$ by "completing the square" method, giving your answers correct to 2 decimal places. [4]

20. Solve the equation $x^2 - 7x - 13 = 0$ by "completing the square" method, giving your answers correct to 2 decimal places. [4]

21. Express $y = 3x^2 - 12x + 7$ in the form $y = a(x + b)^2 + c$. State the values of a, b and c. [4]

22. **(a)** Solve the equation $6x^2 + x - 35 = 0$ by factorisation. [2]

(b) Solve the equation $3x^2 - 6x - 13 = 0$ by completing the square, giving your answers correct to 2 decimal places. [4]

(c) Solve the equation $5x^2 - 14x - 17 = 0$ by using formula. Give your answers correct to 3 significant figures. [3]

23. Solve the following equations, giving your answers correct to 2 decimal places where necessary.

(a) $2x(x - 3) = 3(2x - 5)$ [3]

(b) $\frac{17}{2x - 3} = 3x - 1$ [3]

24. Solve the equation $3x + 9 = \frac{5}{x}$, giving your answers correct to two decimal places. [4]

25. Factorise $2x^2 + 8x + 6$. Hence, write down the prime factors of 286. [3]

26. A car travels from Singapore to Kuala Lumpur, covering a distance of 390 km in a period of x hours. A slow train travels the same distance and it takes 4 hours more to reach the destination.

Write down, in terms of x,

(a) the average speed of the car in km/h, [1]

(b) the average speed of the train in km/h. [1]

(c) If the average speed of the train is 55 km/h faster than the train, form an equation in x and show that it reduces to $11x^2 + 44x - 312 = 0$. [4]

Solve the above equation to find

(d) the average speed of the car for the journey, [3]

(e) the time taken by the slow train to travel from Singapore to Kuala Lumpur, giving your answer correct to the nearest minute. [2]

27. A motorboat can sail at a constant speed of x km/h in still water. When it sails with the current in a river, its speed is increased by $3\frac{1}{2}$ km/h and when it sails against the current, its speed is decreased by $3\frac{1}{2}$ km/h. The boat sails from village A to village B against the current and from village B to village C with the current on its way back. Given that the distance from village A to village B is 12 km, that from village B to village C is 9 km and that the total time taken for the whole journey of 21 km is 75 minutes,

(a) write down expressions, in terms of x, for the time taken by the boat to travel from

(i) village A to village B, [1]

(ii) village B to village C, [1]

(b) form an equation in x and show that it reduces to $20x^2 - 336x = 413$, [4]

(c) solve the above equation giving your answer correct to 2 decimal places and state the time taken for the boat to travel from village B to village C, giving your answer correct to the nearest minute. [4]

28. An aircraft flew a distance of 3800 km from Singapore to Perth in Australia at an average speed of v km/h.

(a) Write down an expression in terms of v for the time taken in hours for the journey. [1]

The aircraft returned by the same route at an average speed of $(v + 50)$ km/h.

(b) Write down an expression in terms of v for the time taken in hours for the return journey. [1]

(c) Given that the difference in time between the two journeys is 20 minutes, form an equation in v and show that it reduces to $v^2 + 50v = 570\,000$. [3]

(d) Solve the above equation, giving your answer correct to 1 decimal place. Hence write down the time taken for the journey from Perth to Singapore, giving your answer correct to the nearest minute. [4]

29. A community club chartered a bus for \$1200 to take a group of people for a sightseeing-cum-shopping trip to Johor. It is agreed that each member of the group pay an equal share of the hire of the bus. The group initially consists of x people.

(a) Write down an expression, in terms of x, for the amount of money each member of the group has to pay initially. [1]

(b) On the day of departure, four members of the group could not make it for the trip. The club decided to contribute \$30 from its fund and each of the remaining members had to pay an additional \$5 in order to cover the cost of \$1200.

(i) Write down an expression, in terms of x, for what each member has to pay when the four members cannot make the trip. [1]

(ii) Form an equation in x and show that it reduces to $x^2 + 2x - 960 = 0$. [3]

(iii) Solve the above equation to find the actual amount each member paid for the trip. [2]

30. Solve the following equations, giving your answer correct to three significant figures where necessary.

(a) $x^2 - 10x + 9 = 0$ (b) $6x^2 + x - 12 = 0$

(c) $2x^2 - x - 10 = 0$ (d) $3x^2 - 22x - 16 = 0$

(e) $x^2 + 7x - 5 = 0$ (f) $x + 1 = \dfrac{3}{4(1-x)}$

(g) $2x^2 - 13x + 7 = 0$ (h) $3x^2 - 11x - 17 = 0$

(i) $4x^2 = 12x + 1$ (j) $5x^2 - 7x = 78$

(k) $5x^2 - 2x = 5x + 9$ (l) $6xy + 8x - 9y = 12$ [36]

31. A craftsman and his apprentice working together can complete a project in 4 days. If each works on the project individually, the apprentice would have taken 6 days more than the craftsman. How long would it take for the apprentice to do the job alone? [6]

32. A water tank can be filled by two pipes together in 6 minutes. If the tank is filled by the pipes individually, it would take the smaller pipe 5 minutes longer than the big pipe. Find the time in which each pipe alone would fill the tank. [6]

33. A man bought a toy car for \$$x$ and sold it for \$78, thus making a profit of $\frac{1}{2}x$%. Find the value of x. [4]

34. A rectangle has a diagonal 15 cm long. If the length of the rectangle is 3 cm longer than its width, find the length of the rectangle. [4]

35. The product of two consecutive even integers plus twice their sum is 164. Find the integers. [4]

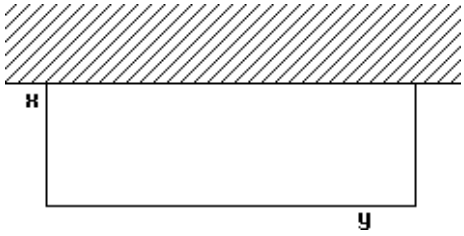
36. The speed of a boat in still water is 12 km/h. The boat travels 18 km upstream and 40 km downstream in a total time of 4 hours 40 minutes. Calculate the speed of the current of the river. [4]

37. The area of a triangle is 15.4 cm^2 and the height is 3.2 cm longer than the base. Find the length of the base. [4]

38. A piece of wire 48 cm long is bent to form the perimeter of a rectangle of area 72 cm^2. Find the lengths of the sides of the rectangle. [4]

39. A rectangular photograph is placed on a sheet of vanguard paper measuring 25 cm by 18 cm. There is a border of uniform width x cm around the photograph. If the total area of the border is 272 cm^2, find the value of x. [4]

40. A farmer uses 80 m of fencing to make three sides of a rectangular enclosure. The fourth side is a straight hedge. Find the length and width of the enclosure if the area enclosed is 600 m^2. [4]



41. If a train had travelled 8 km/h faster, it would have travelled 350 km. Find the original speed of the train, giving your answer correct to the nearest km/h. [5]

42. The hypotenuse of a right-angled triangle is 22 cm and the sum of the other two sides is 30 cm. Find the lengths of the other two sides. [4]

43. A man made a car journey from Johor Bahru to Segamat, a distance of 195 km. For the first 150 km, his average speed was x km/h and for the last 45 km, his average speed was 10 km/h more than that for the earlier part. If the total time taken for the journey was 3 hours 15 minutes, form an equation in x and show that it reduces to $13x^2 - 650x + 6000 = 0$. Solve this equation and hence find the average speed for the last 45 km. [7]

Answers

1. (a) $(a-1)(b-1)$ (b) $(b-c)(b+c+1)$ (c) $6(3+y)(3-y)$

2. (a) $xy(x+2y)(x-2y)$ (b) $(y+x-3)(y-x+3)$

3. (a) $(2a+b)(2a-b)$ (b) $(3x+1)(x-1)$ (c) $\frac{3}{3v+7}$

4. (a) 3 or -3 (b) -1 or 0.5 (c) 3.5 or -4.5

5. (a) 0 or $2\frac{1}{4}$ (b) 2 or 4

6. (a) 1.10 or -2.43 (b) 7.61 or 0.39 (c) no real roots

7. (a) $(x-y-1)(x-y+4)$ (b) $4x(x+2)$ (c) $(2x-1)(3x+5)$
(d) $3a(a+2b)(a-2b)$

8. (a) $2x(x-4)$ (b) $(2x+3y)(x-y)$ (c) $(x-7y)(x+5y)$

9. (a) 12 or -2 (b) 1.90 or -1.23 (c) 1 or -5

10. (a) $\frac{x-3}{3x+4}$ (b) (i) 0 or 3 (ii) $\frac{2}{3}$ or -5 (iii) -2 or 3

11. (a) $1\frac{1}{3}$ or $-1\frac{1}{4}$ (b) ± 4.18 (c) 0 or 0.269

12. (a) $2x(x+4)(x-4)$ (b) $(x-5)(x-7)$; ± 2.24, ± 2.65

13. $k = 9$

14. (a) 20 (b) $(2a-3b+c)(2a+3b-c)$

15. $x-8$, $x^2-24x+143=0$, $AB = 11$ or 13

16. $13\frac{1}{3}$

17. 7.41 or -0.41

18. $7\frac{1}{2}$

19. 1.43 or -5.93

20. 8.52 or -1.52

21. $a = 3, \quad b = -2, \quad c = -5$

22. (a) $2\frac{1}{3}$ or $-2\frac{1}{2}$ (b) 3.31 or −1.31 (c) 3.72 or −0.915

23. (a) 4.22 or 1.78 (b) 1.5 or $\frac{1}{3}$

24. 0.48 or −3.48

25. $2(x + 1)(x + 3)$; 2, 11, 13

26. (a) $\frac{390}{x}$ (b) $\frac{390}{x+4}$ (d) 105.7 km/h (e) 7h 41min

27. (a) (i) $\frac{12}{x-3\frac{1}{2}}$ (ii) $\frac{9}{x+3\frac{1}{2}}$ (c) $x = 17.95$, 25 min

28. (a) $\frac{3800}{v}$ (b) $\frac{3800}{v+50}$ (d) 730.4, 4h 52min

29. (a) $\frac{1200}{x}$ (b) (i) $\frac{1200}{x-4}$ (ii) $\frac{1200}{x} + 5 = \frac{1170}{x-4}$ (iii) $x = 30$; \$45

30. (a) 1, 9 (b) $1\frac{1}{3}, \ -1\frac{1}{2}$ (c) $2\frac{1}{2}, \ -2$ (d) $-\frac{2}{3}, \ 8$

(e) 0.653, −7.65 (f) $\frac{1}{2}, -\frac{1}{2}$ (g) 5.91, 0.593 (h) 4.84, − 1.17

(i) 3.08, − 0.0811 (j) 4.71, − 3.31 (k) 2.21, −0.813 (l) $1\frac{1}{2}, -1\frac{1}{3}$

31. 12 days **32.** 10 min, 15 min **33.** 60 **34.** 12 cm

35. 10, 12 **36.** 3 km/h **37.** 4.37 cm **38.** 3.51, 21.49

39. 3.85 **40.** 30, 10 **41.** 57 km/h **42.** 19.1, 10.9

43. $x = 49.3$ or 0.72 ; 59.3 km/h

Chapter 2

Secondary 3 Mathematics
Chapter 2 Indices and Standard Form

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 32)

$-1 + 9 - 9 + 2 = 1$

$1 \times 9 - 9 + 2 = 2$

$1 + 9 - 9 + 2 = 3$

$1 + 9 \div 9 + 2 = 4$

$1 + \sqrt{9} + \sqrt{9} - 2 = 5$

$1 + 9 \div \sqrt{9} + 2 = 6$

$1 + 9 \div \sqrt{9} \times 2 = 7$

$19 - 9 - 2 = 8$

$1 + \sqrt{9} + \sqrt{9} + 2 = 9$

$(1 + \sqrt{9}) \times \sqrt{9} - 2 = 10$

$1 \times \sqrt{9} \times \sqrt{9} + 2 = 11$

$1 + \sqrt{9} \times \sqrt{9} + 2 = 12$

Thinking Time (pg 39)

The rule $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ is only applicable when a and b are positive integers.

Secondary 3 Mathematics
Chapter 2 Indices and Standard Form

GENERAL NOTES

This will be the first time that pupils will be studying the topic on indices although they would have encountered indices with base 10 in Sec 2. Teachers can initiate discussion regarding the convenience of using indices and the application of this knowledge i.e. for very *large* or very *small* quantities. The mass of the Earth and that of an atom are two examples that students can comprehend easily. Other examples are the number of people on the planet Earth, the number of air molecules in a typical classroom, etc. Teachers may also wish to introduce some of the terms that are used to count extremely large and extremely small numbers such as those found in the British and American systems of numbers. One common difference is the value of 'billion' which is different in the British and American vocabulary although the American version is now commonly adopted.

You may like to introduce this story of how a rich Chinese miser learned to count:

The miser engaged a tutor to teach him how to write numerals. The tutor taught him how to write one, I, then the number two, II, which the miser learnt very quickly and then the number three, III. The miser found all these too simple and so found no necessity to learn further and pay more, so he dismissed the tutor thereafter. One day he wanted to write ten thousand and what a big and long piece of paper he needed!

The index notation is a simple and short representation of the multiplication of the same number.

Exploration on page 41 gives opportunities for students to practise looking for a pattern. Questions of this nature are common in mathematics competitions.

Common Errors Made By Students

Students have learnt the index notation in their primary school days and should hence find the first two laws easy to comprehend. However, many students tend to confuse the rules as they do the exercises.

Some of the common errors involving indices are:

1. $a^2 \times a^3 = a^6$
2. $a^{10} \div a^2 = a^5$
3. $a^3 + a^2 = a^5$
4. $a^8 - a^2 = a^6$
5. $(3^4)^2 = 9^8$
6. $(2x^3)^3 = 2x^9$
7. $2x^{-3} = \dfrac{1}{2x^3}$
8. $\dfrac{3}{a^4} = \dfrac{a^{-4}}{3}$
9. $3\times10^4 + 4\times10^4 = 7\times10^8$
10. $(2a+b)^7 = 14a^7 + b^7$

NE MESSAGES

Page 22 Introduction

As only 689 500 people in Singapore pay income tax (in 2005), this turns up to be less than 20% of the population who are paying tax. Do you know how much tax do your parents pay? What are the other taxes that we have to pay to the government? Where do these tax monies go to? What are they used for?

Part of these monies goes into paying the civil servants and government officers who work in government institutions. A portion of these monies also goes into defence, education and health care. Can you think of other areas where the tax monies could be utilised?

How will the increase of GST from 5% to 7% (w.e.f July 2007) affect the people around you?

Page 49 Exercise 2h Q27

Page 52 Review Questions 2 Q6

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: 35 min

Class: _____ Marks: /14

Secondary 3 Multiple-Choice Questions
Chapter 2 Indices and Standard Form

1. Simplify $(2xy^2)^3(x^2y)^4$.

   (A) $6x^{11}y^{10}$ (B) $6x^{24}y^{24}$ (C) $8x^{11}y^{10}$
   (D) $8x^{24}y^{24}$ (E) $8x^{11}y^{24}$ ( )

2. Simplify $\frac{(2x^2)^4}{3xy^2} \div \frac{x^2}{2y^3}$.

   (A) $\frac{4}{3}x^9y^5$ (B) $\frac{16x^9}{3y^5}$ (C) $\frac{16}{3}x^7y^5$
   (D) $\frac{32}{5}x^5y$ (E) $\frac{64}{3}x^5y$ ( )

3. Simplify $(2xy)^3 \div 2x^2y$.

   (A) $3xy^2$ (B) $4xy^2$ (C) $4x^5y$ (D) $3x^5y^2$ (E) $8xy^2$ ( )

4. Solve the equation $32^x = 16$.

   (A) $\frac{1}{2}$ (B) $\frac{4}{5}$ (C) $1\frac{1}{4}$ (D) $\frac{1}{4}$ (E) Cannot be solved. ( )

5. Simplify $\frac{(x^{-2}y^3)^2}{x^2y^{-1}}$.

   (A) $x^{-2}y^7$ (B) $x^{-6}y^2$ (C) $x^{-6}y^6$ (D) $x^{-6}y^7$ (E) x^6y^4 ( )

6. Simplify $2^{10} \times 3^{10}$.

   (A) 5^{10} (B) 5^{20} (C) 6^{10} (D) 6^{20} (E) 6^{100} ( )

7. Simplify $4^{x+2} \times 8^{2-x}$.

   (A) 12 (B) 12^2 (C) 12^{10-x} (D) 4^{3-x} (E) 2^x ( )

8. Simplify $\frac{10xy^4}{30x^2y^2z^3}$.

   (A) $\frac{xy^2}{3xz^3}$ (B) $\frac{2xy^2}{3x^2z^3}$ (C) $\frac{y^2}{3xz^3}$ (D) $\frac{2y^2}{6xy}$ (E) $\frac{y^2}{3xz^2}$ ( )

9. Evaluate $-(\frac{-1}{8})^{-\frac{2}{3}}$.

(A) 4 (B) -4 (C) $\frac{1}{4}$ (D) $-\frac{1}{4}$ (E) $\frac{-1}{2}$ ( )

10. Solve the equation $8^{\frac{2}{3}} = (24x)^{\frac{1}{2}}$.

(A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $1\frac{1}{2}$ (E) $1\frac{3}{4}$ ( )

11. Simplify $2^{2x} \div 4^{3x} \times 64^{\frac{1}{2}x}$.

(A) 2^x (B) 2^{-x} (C) 4^x (D) 4^{2x} (E) 8^x ( )

12. $(x^{-\frac{1}{3}})^{-3}$ is equal to

(A) $x^{-3\frac{1}{3}}$ (B) $x^{\frac{1}{9}}$ (C) x (D) x^0 (E) x^{-1} ( )

13. Find the value of $-(-\frac{1}{27})^{-\frac{2}{3}}$.

(A) 9 (B) -9 (C) $\frac{1}{9}$ (D) $-\frac{1}{9}$ (E) $\frac{1}{3}$ ( )

14. Solve the simultaneous equations $4^{x+y} = 16$; $3^{x-y} = 81$.

(A) $x = 4,\ y = 0$ (B) $x = 4,\ y = -1$ (C) $x = 3,\ y = 1$
(D) $x = 3,\ y = 0$ (E) $x = 3,\ y = -1$ ( )

Answers

1. C	2. D	3. B	4. B	5. D
6. C	7. C	8. C	9. D	10. C
11. B	12. C	13. B	14. E	

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: 

Secondary 3 Mathematics Test
Chapter 2 Indices and Standard Form

1. Evaluate $2^3 + 3^2 - (1\frac{2}{3})^{-1}$. [2]

2. Simplify $(-2x^3 y^2)^4 \div 8x^2 y^{-3}$. [2]

3. (a) Find the value of $(\frac{1}{2})^{-2} \times (\frac{3}{4})^0 \times (\frac{5}{6})^2$ [2]

   (b) Simplify the following and leave your answer in positive indices only:

   $(2x^{-3}y^2)^{-2}$ $(3x^3 y^{-2})^2$ [2]

4. Evaluate each of the following:

   (a) $(8^0)^{-14} + 2^{-1}$ [2]

   (b) $(\frac{2}{3})^{-1} \times (\frac{2}{5})^{-2} \div \frac{5}{8}$ [2]

5. Evaluate each of the following:

   (a) $4^2 \times 4 \times 4^0$ [1]

   (b) $(\frac{1}{4})^{-2}$ [1]

   (c) $(\frac{2}{3})^{-3} \div (\frac{3}{4})^0$ [1]

6. Simplify (a) $\frac{2^3(-x)^3}{(3x^3)^2}$ [1]

   (b) $(2xy^{-2})^3 \div (4x^{-2}y^3)^2$ [2]

7. Simplify and express your answer in terms of positive indices only:

   (a) $(x^{-8}y^6z^4)^{-\frac{3}{2}}$ [2]

   (b) $(a^2 b^3)^{-3} \div a^{-4}b^{-7}$ [2]

8. Simplify the following, expressing your answer in positive index form:

(a) $\dfrac{x^3y^{-2}}{x^{-1}y^3}$ [1]

(b) $\dfrac{(a^{-2}b^3)^4}{ab^{-4}}$ [2]

9. Simplify the following:

(a) $2m^4 \times 3m^{-5} \div m^{-2}$ [2]

(b) $\dfrac{(2ab)^2}{21a^4b^5} \div \dfrac{8ab^4}{7a^5b^3}$ [2]

10. Given that $a = 2 \times 10^3$ and $b = 4 \times 10^{-5}$, calculate the following and leave your answers in standard form.

(a) ab (b) $\dfrac{a}{b}$ (c) $a + \dfrac{1}{b}$ [6]

11. (a) Express 0.005 724 in standard form.
(b) Evaluate $(8 \times 10^3) \times (3.2 \times 10^{-2})$, giving your answers in standard form. [4]

12. Evaluate the following and leave your answers in standard form.
(a) $3.42 \times 10^8 - 9.6 \times 10^7$
(b) $(5.84 \times 10^{-4}) \div (2.0 \times 10^{-15})$ [4]

13. Evaluate the following, giving your answers in standard form.
(a) $7\ (1.23 \times 10^{-4})$
(b) $0.46 \times 10^5 + 75.8 \times 10^4$ [4]

14. Given that $a = 6 \times 10^8$ and $b = 4 \times 10^6$, find the value of each of the following in standard form.

(a) $\dfrac{3a}{2b}$ (b) a – 3b [4]

15. Given that $x = 2.8 \times 10^{-5}$ and $y = 7 \times 10^3$, find (a) $5xy^2$ (b) $\dfrac{2y}{x}$, leaving your answers in standard form. [4]

16. (a) Rewrite 84.37×10^{-4} as a decimal.
(b) Express 8.3674×10^4 in ordinary notation, correct to the nearest thousand. [3]

17. The population of Singapore is recorded as 3 947 000 in 2005. Express 0.000 045 23 cm in standard form correct to 2 significant figures. [2]

18. The radius of a micro-organism is 0.000 045 23 cm. Express 0.000 045 23 cm in standard form correct to 3 significant figures. [2]

19. Given that $p = 9.5 \times 10^{7}$ and $q = 5.0 \times 10^{-6}$, calculate, expressing each answer in standard form, the value of

    (a) $2pq$ (b) $\frac{p}{4q}$ [4]

20. Evaluate each of the following:

    (a) $(4.2 \times 10^{7}) \times (2.5 \times 10^{-3})$ (b) $(8.74 \times 10^{5}) + (8.6 \times 10^{4})$

    (c) $(6.4 \times 10^{5}) \div (20 \times 10^{-3})$ [4]

21. Given that $x = 6 \times 10^{3}$ and $y = 5 \times 10^{-4}$, calculate the following and leave your answers in standard form.

    (a) $\frac{x}{y}$ (b) $x + \frac{2}{y}$ [4]

22. If $A = 3.4 \times 10^{7}$ and $B = 0.374 \times 10^{9}$, find the value of $\frac{A}{B - A}$, giving your answers in standard form. [4]

23. A rectangular field measures 4.5×10^{2} m by 3.6×10^{2} m. Calculate its

    (a) area, (b) perimeter, giving your answers in standard form. [4]

24. If the area of a circle is 2.54×10^{6} cm, find the

    (a) radius, (b) perimeter of the circle, giving your answers in standard form correct to 3 significant figures. (Take $\pi = 3.142$) [4]

25. The population of Singapore was recently estimated to be three million, eight hundred and eighty thousand.

    (a) Write the number in standard form.

    (b) The total land area of Singapore is approximately 640 km^2. Calculate the average number of people per square kilometer of the land area, giving your answer correct to 2 decimal places. [4]

26. Express the following in standard form:
(a) 324 kg in g, (b) 1.2 km/min in cm/s. [4]

27. Evaluate $\frac{3^4 \times 6^{-5} \times 5^{-6}}{4 \times 10^{-5}}$ without using a calculator. [3]

28. Simplify $\frac{(-2x^3y^{-4})^3(xy^{-1})^{-2}}{(4x^{-2}y^{-3})^2}$. [3]

29. Simplify $a^{n+1}b^3 \div \frac{a^{n+4}}{a^2b^{-5}}$. [2]

30. Simplify $\frac{(-2x^3y)^2}{6xy^3}$ giving your answer in positive indices. [2]

31. Evaluate (a) $7^{1\frac{1}{2}} \times 7^{\frac{3}{4}} \div 7^{1\frac{1}{4}}$ [2]

(b) $\frac{4^{-2} \times 7^{-1} \times 4^4}{7^{-3} \times 4^3}$ [2]

32. Evaluate each of the following, simplifying your answers as far as possible:

(a) $(1997)^1 + (\frac{1}{1997})^0 - (\sqrt{1997})^0$ [1]

(b) $(\frac{3}{5})^0 + (\frac{8^4}{2^4}) - (\frac{27^3}{9^3})$ [2]

(c) $\frac{4^2 \times 5^7 \times 3^7}{15^7}$ [2]

33. Simplify the following:

(a) $(3a^4 b^7)(5a^{-1}b^6)$ [1]

(b) $\frac{72m^{-1}n^3}{288m^3n^{-4}}$ [1]

34. Simplify $(\frac{x^2y^{-3}}{x^{-5}y^2})^2 \times (\frac{x^{-3}y^{-1}}{x^2y^3})^{-3}$, giving your answer in positive indices. [3]

35. Solve the equation $5^{2x-3} = \frac{1}{25}$. [2]

36. Find the value of x when $6^{x} \times 36^{2x-5} = 1$. [2]

37. Solve the equation $9^{2x-5} = 1$. [2]

38. Given that $3^{x} = 5$ and $3^{y} = 7$, find the value of 3^{4x-2y}. [3]

39. Solve the equations:
(a) $2^{x} \times 4^{x+2} \times 8^{x-1} = 64$ [2]
(b) $5^{x} \div 25^{x-1} = 125$ [2]

40. Solve the following equations:
(a) $2^{x} \times 4^{x-1} = 16$ [2]
(b) $(\frac{1}{3})^{x} \div 9^{x} = 81^{x+2}$ [3]

41. Find the value of x given that $4 \times 3^{2x-1} = 108$. [2]

42. (a) Simplify $7^{2x+1} - 4(7^{2x})$. [1]
(b) Use the result from (a) or otherwise, and solve the equation $7^{2x+1} - 4(7^{2x}) = 1029$. [2]

43. Solve the equation $2^{x} \times 4^{x+1} \div 8^{3x-4} = \frac{1}{32}$. [3]

44. Evaluate $(0.1)^{-2} \times 0.2^{2}$ [2]

45. (a) Evaluate $(0.027)^{-\frac{1}{3}} + 16^{0.75} + \left(\sqrt{\frac{1}{\sqrt{2}-1}}\right)^{0} + (-3)^{-1}$. [2]

(b) Simplify $(16a^{4})^{-\frac{1}{4}} \div (0.001a^{6})^{\frac{1}{3}}$. [2]

46. (a) Evaluate

(i) $\left(\frac{256}{81}\right)^{\frac{1}{4}}$ (ii) $\left(\frac{2}{3}\right)^{-4}$ [2]

(b) Solve the equation $27^{x} = \sqrt{9}$ [1]

47. Evaluate each of the following.
(a) $6^{5.5} \div 6^{4.5}$ [1]
(b) $2^{\frac{1}{2}} \times 4^{\frac{1}{2}} \times 8^{\frac{1}{2}}$ [2]

48. Evaluate
(a) $2^3 + (32)^{-\frac{1}{5}} + \left(\frac{2}{3}\right)^0$ (b) $4^{\frac{1}{3}} \times 4^{1\frac{2}{3}}$ [2]

49. Evaluate
(a) $\left(\frac{2}{3}\right)^{-2}$ (b) $8^{\frac{5}{3}}$ (c) $\sqrt{5} \times 5^{\frac{3}{2}}$ [3]

50. Simplify $\frac{45x^4y^3}{4z^5} \div \frac{15x^3y^5}{8x^2z^3}$. [2]

51. Given that $x^{2y} = 3$, find the value of $3x^{6y} - 9$. [2]

52. Simplify each of the following.
(a) $(a^{\frac{1}{2}} b^2)^{\frac{3}{4}} \times (a^2 b^{-4})^{-\frac{1}{4}}$ [2]
(b) $\sqrt[3]{125x^9} \div (81x^{-4})^{\frac{1}{2}}$ [2]

53. (a) Given that $9^{2x} = \sqrt{3}$, find the value of x [2]
(b) Express $x^2\left(\frac{x^{-3}}{y^4}\right)^{-2}$ with positive indices. [2]
(c) Simplify $\frac{(a^3b^{-3})^{-2}}{ab}$ and express your answer with negative indices. [2]

54. (a) Solve the equation $9^x = \frac{1}{\sqrt{27}}$. [2]
(b) Simplify $\frac{a^{\frac{5}{4}} \sqrt[3]{a^4}}{a^{-3}}$ giving your answer with positive index. [2]

55. Evaluate

(a) $32^{-\frac{4}{5}}$ [1]

(b) $2^{-3} \times \left(\frac{9}{4}\right)^{-1\frac{1}{2}} \times \left(7\frac{1}{2}\right)^{0}$. [2]

56. Evaluate each of the following.

(a) $\left(\frac{1}{7}\right)^{0}$ (b) $(0.14)^{2}$ (c) $64^{-\frac{1}{3}}$ [3]

57. Simplify $\frac{3a^{\frac{1}{4}} \times 2a^{-\frac{1}{2}}}{12a^{-2}}$. [2]

58. (a) Evaluate $16^{\frac{3}{4}} + \left(\frac{2}{3}\right)^{-2}$. [2]

(b) Given that $x^{-3} = 4$, find the value of x^3. [1]

59. (a) Simplify $4x^5 \times 5x^4$. [1]

(b) Find the smallest integer value of x for which $3^x > 10$. [2]

(c) Express $\frac{2x-3}{6} - \frac{5x-1}{3} + \frac{1}{4}$ as a single fraction in its lowest terms. [2]

60. Simplify the following and leave your answer in positive indices:

(a) $(x^{-6})^{\frac{1}{2}}$ [1]

(b) $(x^{-12}y^{36})^{-\frac{2}{3}}$ [2]

61. Simplify each of the following, giving your answer in positive indices only. [8]

(a) $x^3 \times x^2 \div x^{-4}$ (b) $y^2 \div y^3 \times y^7$

(c) $2a^2 \times 5a^3$ (d) $5a^3 \times 2a^{-3} \div a^4$

(e) $6a^2 \times (2a)^3 \div 4a$ (f) $(2p^{-2}q^3) \div 4pq$

(g) $2(pq^{-2})^4 \div 4q^{-1}$ (h) $(a^2)^{-3} \times a^4 \div a^{-1}$

62. Simplify each of the following, giving your answer in positive indices only. [4]

(a) $\frac{8a^3b^2 \times 4a}{(2ab)^3}$ (b) $\frac{(3xy)^2 \div 4x^2y}{(2xy)^3 \div 8xy^3}$

63. Simplify each of the following, giving your answer in negative indices only. [4]

(a) $x^{-4} \times x^{-5} \div x^{-6}$ (b) $a^7 \div a^{-2} \times a^{-4}$

(c) $(m^4 \div m^{-1})^{-2}$ (d) $(2d^{-4})^3 \div 4d^{-1}$

64. Simplify each of the following, giving your answer in negative indices only. [6]

(a) $(7a^4 \times 2a^3)^2 \div 14a^5$ (b) $(ab^{-4})^5 \div a^{-1}b^{-5}$

(c) $\dfrac{a^3b \times (2ab^4)^4}{4a^{-1}b^{-4}}$ (d) $\dfrac{3a(2b)^3 \div 8ab}{2a^3 \times (3b)^3}$

65. Simplify each of the following, giving your answer in negative indices only. [4]

(a) $\dfrac{a^2 \times (ab^3)^6}{(2ab^4)^{-1} \times 8a^{-4}}$ (b) $\dfrac{(-2xy)^2 \div 4x^3y^2}{(4x^2y)^{-2} \times x^5y^6}$

66. Express each of the following as a fraction or an integer. [6]

(a) $2^{-3} \times 5^2$ (b) $23^0 \div 3^{-3} \times 2^4$

(c) $4^{-2} \times 8^{-1} \div 16^{-2}$ (d) $(1\frac{1}{2})^{-2} \times (\frac{3}{4})^2 \div (-2)^{-2}$

67. Express each of the following as a fraction or an integer. [12]

(a) $10^{-1} \times 5^2 \div 6^{-2}$ (b) $4^{-3} \div (5)^{-2} \div (7\frac{1}{2})^0$ (c) $(3^{-2})^2 \div (4^{-1})^2$

(d) $(-2)^3 \div (-3)^{-2}$ (e) $(2^{-3})^4 \div (8^{-1})^2$ (f) $7^2 \times 49^{-3} \div (\frac{1}{7})^{-4}$

68. Solve the following equations: [4]

(a) $3^x = 243$ (b) $2^x = \dfrac{1}{64}$

(c) $23^x = 1$ (d) $x^7 = -1$

69. Solve the following equations: [6]

(a) $5x^4 = 405$ (b) $27x^3 = 1$ (c) $5^x = \dfrac{1}{125}$

70. Solve the following equations: [6]

(a) $x^{-2} = 36$ (b) $2^3 \times 8^x = 0.25$ (c) $2^{10} \div 4 = 2^x$

71. Simplify the following, giving your answer with positive indices. [4]

(a) $(\dfrac{a^2}{b^{-2}})^{-1}$ (b) $\dfrac{(xy^3)^{-1}}{(x^{-1}y^2)^{-3}}$

72. Simplify the following, giving your answer with negative indices. [6]

(a) $\dfrac{p^5q^6}{q^{-3}p^{-1}} \times \dfrac{p^{-4}q^{-5}}{p^2q^3}$ (b) $\dfrac{abc^{-1}}{(a^{-2}b)^2} \times \dfrac{a^2b}{(a^{-1}c^{-3})^{-2}}$

73. Simplify the following expressions. [4]

(a) $(2x^{\frac{1}{2}}) \times (6x^{\frac{3}{2}})$ (b) $5x^{-4\frac{1}{2}} \div 4x^{-1\frac{1}{4}}$

(c) $(2a^{-1})^4 \div (8a^{-\frac{1}{2}})^{\frac{4}{3}}$ (d) $3a^{-2} \div (27a)^{\frac{2}{3}}$

74. Evaluate the following: [3]

(a) $169^{\frac{3}{2}}$ (b) $100^{-\frac{1}{2}}$ (c) $(-8)^{\frac{2}{3}}$

75. Evaluate the following: [6]

(a) $(3\frac{3}{8})^{\frac{1}{3}} \div (\frac{1}{8})^{-1}$ (b) $(\frac{1}{64})^{-\frac{1}{3}} + (-3)^{-2}$ (c) $(1-\frac{1}{2})^{-1} \div (2\frac{1}{4})^{-\frac{1}{2}}$

76. Solve the following equations. [5]

(a) $x^7 = 7^0$ (b) $5^{-\frac{2}{3}} \div \sqrt{5} = 5^x$ (c) $4^x = 0.125$

77. Solve the following equations. [6]

(a) $8^{2x+1} = \sqrt{32}$ (b) $10^{5x-1} = 0.001$ (c) $3^{x-1} \times 9^{x+3} = 27^{2x-4}$

78. Solve the following equations. [6]

(a) $4^{2x-1} = 8^{x+3}$ (b) $16^{\frac{1}{2}x+3} = 8^{x+1}$

79. Given that $a = 4.2 \times 10^5$ and $b = 8.3 \times 10^4$, find the value of the following, expressing your answer in standard form. [4]

(a) $a + b$ (b) $a - b$ (c) ab (d) $\frac{a}{b}$

80. Given that $x^{-3} = 4$, find the value of x^3. [2]

81. If $p^{-2} = 5q^{\frac{1}{3}}$, calculate the value of [4]

(a) p when $q = 125$, (b) q when $p = \frac{2}{5}$.

82. Given that $x = 1.2 \times 10^6$, evaluate $\sqrt{x+10^4}$. [2]

83. Given that $(ab)^{-2} = x^{\frac{1}{2}}$, find the value of x when $a = \frac{2}{5}$ and $b = 3\frac{3}{4}$. [3]

84. Evaluate each of the following without the use of a calculator, giving your answer in standard form correct to 4 significant figures. [10]

(a) $3.12 \times 10^4 + 2.6 \times 10^2$
(b) $4.76 \times 10^4 - 6.13 \times 10^3$
(c) $7.91 \times 10^9 + 6.14 \times 10^8$
(d) $3.24 \times 10^8 - 9.86 \times 10^7$
(e) $1.02 \times 10^{-5} + 3.19 \times 10^{-6}$
(f) $8.59 \times 10^{10} + 16.7 \times 10^9$
(g) $5.48 \times 10^{-8} - 76.4 \times 10^{-6}$
(h) $324 \times 10^6 - 1.86 \times 10^7$
(i) $76.34 \times 10^5 + 183.4 \times 10^4$
(j) $36.8 \times 10^{18} - 485 \times 10^{15}$

85. Use your calculator to evaluate each of the following, giving your answer in standard form correct to 4 significant figures. [10]

(a) $3.18 \times 10^4 \times 6.45 \times 10^2$
(b) $4.59 \times 10^{-3} \times 8.674 \times 10^7$
(c) $5.43 \times 10^9 \div (3.27 \times 10^8)$
(d) $3.58 \times 10^{-10} \div (7.61 \times 10^{-9})$
(e) $4.95 \times 10^{-5} \div (3.14 \times 10^{-6})$
(f) $6.45 \times 10^2 \div (3.27 \times 10^7)$
(g) $32.65 \times 10^{-8} \times 4.59 \times 10^7$
(h) $5.149 \times 10^7 \times 3.26 \times 10^{-4}$
(i) $34.95 \times 10^5 \times 672.6 \times 10^4$
(j) $19.79 \times 10^8 \div (39.76 \times 10^{-3})$

Answers

1. $16\frac{2}{5}$

2. $2x^{10}y^{11}$

3. (a) $2\frac{7}{9}$
   (b) $\frac{9x^{12}}{4y^{8}}$

4. (a) $1\frac{1}{2}$
   (b) 15

5. (a) 64
   (b) 16
   (c) $6\frac{3}{4}$

6. (a) $\frac{-8}{9x^3}$
   (b) $\frac{x^7}{2y^{12}}$

7. (a) $\frac{x^{12}}{y^9 z^6}$
   (b) $\frac{1}{a^2b^2}$

8. (a) $\frac{x^4}{y^5}$
   (b) $\frac{b^{12}}{a^9}$

9. (a) $6m$
   (b) $\frac{a^2}{6b^4}$

10. (a) 8×10^3
    (b) 5×10^7
    (c) 2.7×10^4

11. (a) 5.724×10^{-3}
    (b) 2.56×10^2

12. (a) 2.46×10^8
    (b) 2.92×10^{11}

13. (a) 8.61×10^{-4}
    (b) 8.04×10^5

14. (a) 2.25×10^2
    (b) 5.88×10^8

15. (a) 6.86×10^3
    (b) 5×10^8

16. (a) 0.000 843
    (b) 84 000

17. 3.9×10^6

18. 4.52×10^{-5}

19. (a) 9.5×10^2
    (b) 4.75×10^{12}

20. (a) 1.05×10^5
    (b) 9.6×10^5
    (c) 3.2×10^7

21. (a) 1.2×10^7
    (b) 1.0×10^4

22. 1.0×10^{-1}

23. (a) 1.62×10^5 m^2
    (b) 1.62×10^3 m

24. (a) 8.99×10^2 cm^2
    (b) 5.65×10^3 cm

25. (a) 3.88×10^6
    (b) 6.06×10^3

26. (a) 3.24×10^5
    (b) 2.0×10^3

27. $\frac{1}{60}$

28. $\frac{-x^{11}}{2y^4}$

29. $\frac{1}{ab^2}$

30. $\frac{2x^5}{3y}$

31. (a) 7
    (b) $12\frac{1}{4}$

32. (a) 1997
    (b) 230
    (c) 16

33. (a) $15a^3 b^{13}$
    (b) $\frac{n^7}{4m^4}$

34. $x^{29}y^2$

35. $\frac{1}{2}$

36. 2

37. $2\frac{1}{2}$

38. $12\frac{37}{49}$

39. (a) $1\frac{1}{6}$
    (b) -1

40. (a) 2
    (b) $-1\frac{1}{7}$

41. $x = 2$

42. (a) $3(7^{2x})$
    (b) $x = 1.5$

43. $3\frac{1}{6}$

44. 4

45. (a) 12
    (b) $\frac{5}{a^3}$

46. (a) (i) $1\frac{1}{3}$
    (ii) $5\frac{1}{16}$
    (b) $\frac{1}{3}$

47. (a) 6
    (b) 8

48. (a) $9\frac{1}{2}$
    (b) 16

49. (a) $2\frac{1}{4}$
    (b) 32
    (c) 25

50. $\frac{6x^3}{z^2y^2}$

51. 72

52. (a) $a^{-\frac{1}{8}}\ b^{2\frac{1}{2}}$
    (b) $\frac{5}{9}\ x^5$

53. (a) $\frac{1}{8}$
    (b) $(xy)^8$
    (c) $\frac{a^{-7}}{b^{-5}}$

54. (a) $-\frac{3}{4}$
    (b) $a^{5\frac{7}{12}}$
    (c) $x = \frac{y-a}{1+k^2}$

55. (a) $\frac{1}{16}$
    (b) $\frac{1}{27}$

56. (a) 1
    (b) 0.0196
    (c) $\frac{1}{4}$

57. $\frac{1}{2}\ a^{1\frac{3}{4}}$

58. (a) $10\frac{1}{4}$
    (b) $\frac{1}{4}$

59. (a) $20x^9$
    (b) $x = 3$

60. (a) $\frac{1}{x^3}$
    (b) $\frac{x^8}{y^{24}}$

61. (a) x^9 (b) y^6 (c) $10a^5$ (d) $\frac{10}{a^4}$ (e) $12a^4$ (f) $\frac{2q^8}{p^7}$

(g) $\frac{p^4}{2q^7}$ (h) $\frac{1}{a}$

62. (a) $\frac{4a}{b}$ (b) $\frac{9y}{4x^2}$

63. (a) x^{-3} (b) $\frac{1}{a^{-5}}$ (c) m^{-10} (d) $2d^{-11}$

64. (a) $\frac{14}{a^{-9}}$ (b) $\frac{b^{-15}}{a^{-6}}$ (c) $\frac{4}{a^{-8}b^{-21}}$ (d) $\frac{a^{-3}b^{-1}}{18}$

65. (a) $\frac{1}{a^{-13}b^{-22}}$ (b) $16x^{-2}y^{-5}$

66. (a) $3\frac{1}{8}$ (b) 432 (c) 2 (d) 1

67. (a) 90 (b) $\frac{25}{64}$ (c) $\frac{16}{81}$ (d) -72 (e) $\frac{1}{64}$ (f) 1

68. (a) 5 (b) -6 (c) 0 (d) -1

69. (a) ± 3 (b) $\frac{1}{3}$ (c) -3

70 (a) $\pm\frac{1}{6}$ (b) $-1\frac{2}{3}$ (c) 8

71. (a) $\frac{1}{a^2b^2}$ (b) $\frac{y^3}{x^4}$

72. (a) $\frac{1}{q^{-1}}$ (b) $\frac{c^{-7}}{a^{-5}}$

73. (a) $12x^2$ (b) $\frac{5}{4}x^{-3\frac{1}{4}}$ (c) $a^{-3\frac{1}{3}}$ (d) $\frac{1}{3}a^{-1\frac{1}{3}}$

74. (a) 2197 (b) $\frac{1}{10}$ (c) 4

75. (a) $\frac{3}{16}$ (b) $4\frac{1}{9}$ (c) 3

76. (a) 1 (b) $-1\frac{1}{6}$ (c) $-1\frac{1}{2}$

77. (a) $-\frac{1}{12}$ (b) $-\frac{2}{5}$ (c) $5\frac{2}{3}$

78. (a) 11 (b) 9

79. (a) 5.03×10^5 (b) 3.37×10^5 (c) 3.486×10^{10} (d) 1.976×10^{-1}

80. $\frac{1}{4}$

81. $\frac{1}{5}$

82. 1.1×10^3

83. $\frac{16}{81}$

84. (a) 3.146×10^4 (b) 4.147×10^4 (c) 8.524×10^9 (d) 2.254×10^8
(e) 1.339×10^{-5} (f) 1.026×10^{11} (g) -7.635×10^{-5} (h) 3.054×10^8
(i) 9.468×10^6 (j) 3.632×10^{19}

85. (a) 2.051×10^8 (b) 3.981×10^5 (c) 1.661×10^2 (d) 4.704×10^{-2}
(e) 1.576×10^9 (f) 1.972×10^{-5} (g) 1.449×10^{13} (h) 1.679×10^4
(i) 2.351×10^7 (j) 4.977×10^{10}

Chapter 3

Secondary 3 Mathematics
Chapter 3 Linear Inequalities

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 55)

1.
$$\begin{array}{r} 6\,8\,4\,1\,1 \\ +\quad 2\,9\,0\,4 \\ \hline 7\,1\,3\,1\,5 \end{array}$$

2. A = 1, B = 4, C = 2, D = 8, E = 5, F = 7

Just For Fun (pg 57)

$\left(\sqrt{10}+\sqrt{29}\right)^2 = 10+29+2\sqrt{10}\sqrt{29} = 39+2\sqrt{290}$ but $17^2 = 289$ and $73 = 39 + 2 \times 17$
$\therefore 39+2\sqrt{290} > 39+2\times 17$. Thus $(\sqrt{10}+\sqrt{29})^2 > 73$ and $\sqrt{10}+\sqrt{29} > \sqrt{73}$

Just For Fun (pg 59)

Take a cap from the one labelled "black and white". If the cap taken is say, black, then we know that both caps in the bag must be black. Now that we have identified one bag, we can tell the contents of the bag labelled "white" to be the bag with one black and one white cap and the last bag containing white caps only.

Just For Fun (pg 60)

The 8 buns can be divided into 24 parts. The first traveller originally has 15 parts and the second traveller has 9 parts. The two travellers each ate 8 parts and the Arab ate the other 8 parts, thus the first traveller had given the Arab 7 parts and the second traveller had only given 1 part. Thus the first traveller must be entitled to 7 gold coins and the second traveller to get 1 gold coin.

Just For Fun (pg 62)

$1 + 5 + 5 + 7 = 2 + 4 + 4 + 8$
$1^2 + 5^2 + 7^2 = 2^2 + 4^2 + 4^2 + 8^2$
$7 + 11 + 11 + 13 = 8 + 10 + 10 + 14$

$1 + 5 + 5 + 7 + 8 + 10 + 10 + 14 = 2 + 4 + 4 + 8 + 7 + 11 + 11 + 13$
$1^2 + 5^2 + 5^2 + 7^2 + 8^2 + 10^2 + 10^2 + 14^2 = 2^2 + 4^2 + 4^2 + 8^2 + 7^2 + 11^2 + 11^2 + 13^2$

$1^3 + 5^3 + 5^3 + 7^3 + 8^3 + 10^3 + 10^3 + 14^3 = 2^3 + 4^3 + 4^3 + 8^3 + 7^3 + 11^3 + 11^3 + 13^3$ is correct.

Secondary 3 Mathematics
Chapter 3 Linear Inequalities

GENERAL NOTES

This chapter is moved from Secondary 2 to Secondary 3. It will be new to the Secondary 3 pupils. It provides a refreshing change from solving equations. The teacher may like to discuss the various examples of inequality in real life situations. There is inequality in every society, every family and every organization. Some people are born with a silver spoon in the mouth while others are not so fortunate. Some are born physically stronger than others and others are more intellectually inclined than their friends, etc. It could develop into a lively scene if teachers encourage students to name and discuss the many inequalities and social injustices in life.

After introducing the inequality signs, teachers may like to ask the pupils to find out from the library or the internet as an extra exercise, the person in history who first introduces these signs.

One way of introducing inequalities is by using concrete examples to lead pupils to arrive at the desired result. We know that $5 > 3$. Is $5 + 2 > 3 + 2$? Pupils normally will be quick to respond with an affirmative answer to which the teacher can proceed.

From the above, we see that the inequality is still true when we add a positive number to both sides of an inequality. If $x > y$, then $x + a > y + a$ where a is a positive number.

(Use the same technique to introduce subtraction, multiplication and division of positive numbers to an inequality.)

We know that $5 > 3$. Is $5 – 2 > 3 – 2$?
Thus if $x > y$, then $x – a > y – a$ where a is a positive number.

We know that $7 > 4$. Is $7 \times 3 > 4 \times 3$?
Thus if $x > y$, then $xa > ya$ where a is a positive number.

We have $8 > 4$. Is $\frac{8}{2} > \frac{4}{2}$?
Thus, if $x > y$, then $\frac{x}{a} > \frac{y}{a}$ where a is a positive number.

After introducing the above, the pupils may be asked to have some practice on the use of the above where only multiplication and division of positive numbers are involved.

The teacher may now introduce the concept that when we multiply or divide both sides of an inequality by a negative number, we must change the inequality sign.

We have $8 > 4$. Is $8 \times (–2) > 4 \times (–2)$? Is $\frac{8}{–2} > \frac{4}{–2}$?

Thus, if $x > y$, then $x(a) < y(a)$ and $\frac{x}{a} < \frac{y}{a}$ where a is negative.

The pupils should be asked to work some sums based on the above rules before the short–cut method is introduced, i.e. the inequality sign may be treated as an equal sign where a single term may be transferred from the L.H.S. to the R.H.S. by changing the sign of the term.

Common Errors Made By Students

The most common error made by students is when an inequality is multiplied or divided by a negative number, they tend to forget to change the inequality sign.

One way of overcoming the above is to ask the pupils to transfer the unknown terms to one side, so that later on only multiplication and division by positive numbers are involved.

For example,	may be done as follows:
$3x - 5 > 6x + 4$	$3x - 5 > 6x + 4$
$3x - 6x > 4 + 5$	$-5 - 4 > 6x - 3x$
$-3x > 9$	$-9 > 3x$
$x < -3$	i.e. $x < -3$

NE MESSAGES

We must uphold meritocracy and incorruptibility.

Page 61 Example 6,
Page 63 Exercise 3c Q1,
Page 70 Review Questions 3 Q10

Singapore practises meritocracy. All pupils who do well in their examinations, are rewarded irrespective of their race and religion. The government sets up the Edusave Endowment Fund in 1993 to fund children's education and encourage them to do well in school. For secondary pupils, the top 5% of the pupils in each stream (Special, Express, Normal Academic and Normal Technical) in every school will receive $500 and the next 5% will receive $300 irrespective of their family's income. The next 15% of the pupils in each stream in each school will be given a chance to apply for the Edusave Merit Bursary which will be administered by the Community Development Council and Citizens Consultative Committee. Only pupils from families whose total family income is less than $3000 per month are eligible to apply. Each successful applicant will receive $250. The Good Progress Award is given to pupils who have shown great improvement in their grades in the current year. Each secondary school pupil is rewarded with $150. Thus many pupils are encouraged to do well in their respective streams. This system awards the brightest and also encourages the less academically inclined pupils to work hard.

We must preserve racial and religious harmony.

Page 63 Exercise 3c Q2

Singapore is a multi-racial and multi-religious society consisting of many races and religions. We must work hard to preserve racial and religious harmony. The Chinese Buddhist Lodge has done a good deed in extending a helping hand to the Malay Muslim community. This manifestation of compassion for people from other races and faith goes a long way to foster the cohesiveness of the Singapore society. Through many races, religions, languages and culture, we pursue our destiny. Teachers can elaborate on the many racial and religious conflicts that are happening in many parts of the world. We must not take racial and religious harmony for granted but make an effort to foster better understanding of each other's religion and culture.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: 35 min

Class: _____ Marks: /16

Secondary 3 Multiple-Choice Questions
Chapter 3 Linear Inequalities

1. Solve the inequality: $2x + 3 > 5x - 7$.

   (A) $x < 3$ (B) $x < 3\frac{1}{3}$ (C) $x > 3\frac{1}{3}$

   (D) $x > 3$ (E) None of the above. ( )

2. If $\frac{1}{5} < \frac{1}{x}$ and $x < 0$, then

   (A) $0 < x < \frac{1}{5}$ (B) $-5 < x < 0$ (C) $0 < x < 5$

   (D) $x < -5$ (E) $-\frac{1}{5} < x < 0$ ( )

3. Simplify the inequality $2y - 5 > 2x + 4y + 3$.

   (A) $y - x > 4$ (B) $y - x < x$ (C) $y + x + 4 > 0$

   (D) $y + x + 4 < 0$ (E) $y + x < 1$ ( )

4. Solve the inequality $2x - 3 > 3x - 10$.

   (A) $x > 7$ (B) $x < 7$ (C) $x > -7$ (D) $x < -7$ (E) $x < \frac{13}{5}$ ( )

5. If $3x - 4 > 5x - 17$, one possible value of which is prime is

   (A) 3 (B) 7 (C) 11 (D) 13 (E) 17 ( )

6. The largest integral value of x satisfying the inequality $3x + 7 \geq 7x - 54$ is

   (A) 6 (B) 14 (C) $15\frac{1}{4}$ (D) 15 (E) 16 ( )

7. The smallest integral value of x satisfying the inequality $5x - 7 \geq 2x - 21$ is

   (A) $-4\frac{2}{3}$ (B) 4 (C) 5 (D) –4 (E) –5 ( )

8. Which of the following is/are true?

   (I) $-3 > -2$ (II) $-2 > -3$ (III) $0 < -2$

   (A) I only (B) II only (C) III only

   (D) II and III only (E) I and III only ( )

9. If $x + 6 > 3$ and $2x - 3 < 7$, then

(A) $3 < x < 5$ (B) $-5 < x < -3$ (C) $-3 < x < 10$

(D) $-3 < x < 2$ (E) $-3 < x < 5$ ( )

10. If $\frac{1}{x} < 5$ and $\frac{1}{x} > 3$, then

(A) $3 < x < 5$ (B) $x < \frac{1}{5}$ and $x > \frac{1}{3}$ (C) $\frac{1}{5} < x < \frac{1}{3}$

(D) $-5 < x < -3$ (E) None of the above. ( )

11. If $a > 0$ and $b < 0$, which of the following is true?

(A) $a + b > 0$ (B) $a - b < 0$ (C) $a^2 - b^2 < 0$

(D) $a \div b < 0$ (E) $ab > 0$ ( )

12. If $a > c$ and $b > c$, then

(A) $a > b$ (B) $b > c$ (C) $\frac{a}{b} > 1$

(D) $\frac{a}{b} < 1$ (E) None of the above. ( )

13. Given that $1 < x < 5$ and $-4 < y < 2$, then the greatest value of $\frac{x}{y}$ is

(A) $-\frac{5}{4}$ (B) -4 (C) $\frac{5}{2}$

(D) $-\frac{1}{4}$ (E) Not possible to find. ( )

14. Given that $0 < x < 1$, which of the following expressions will be greatest?

(A) $x + x^2 + x^3$ (B) $\frac{3}{x}$ (C) $\frac{3}{x^2}$

(D) $x + 3x + \frac{3}{x}$ (E) $x^4 + x^2 + \frac{1}{x^2}$ ( )

15. Which of the following is/are true?

(I) $-\pi > -3$ (II) $(-\pi)^2 > (-3)^2$ (III) $2^2 > (-3)^2$

(A) I only (B) II only (C) I and II only

(D) II and III only (E) I and III only ( )

16. Solve the inequality $3 - \frac{2x-7}{4} \leq \frac{x+1}{2} - \frac{3(x+4)}{4}$.

(A) $x \geq -5$ (B) $x \geq 29$ (C) $x \leq 28$ (D) $x \leq -5$ (E) $x \geq 9$ ( )

Answers

1. B	2. B	3. D	4. B
5. A	6. D	7. D	8. B
9. E	10. C	11. D	12. E
13. E	14. C	15. C	16. B

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: 

Secondary 3 Mathematics Test
Chapter 3 Linear Inequalities

1. Fill in the blanks for each of following:
   (a) If $-6 \le 2x \le 8$ then _____ $\le x \le$ _____ . [1]
   (b) If $-2 \le \frac{x}{2} \le 4$ then _____ $\le x \le$ _____ . [1]
   (c) If $-9 \le -3x \le 15$ then _____ $\le x \le$ _____ . [1]
   (d) If $-1 \le -\frac{x}{2} \le 3$ then _____ $\le x \le$ _____ . [1]

2. Given that $-4\frac{2}{3} \le 2k \le 17\frac{1}{3}$, write down
   (a) the smallest integer value of k, [1]
   (b) the largest prime value of k, [1]
   (c) the largest rational value of k. [1]

3. Given that x is an integer, find the largest possible value of x which satisfies the following inequality:
   $$6 - x \ge \frac{2}{3}(x - 8)$$ [2]

4. Given that $4x - 3 \le \frac{1}{3}(2x + 22)$, state the greatest possible value of x if x is
   (a) a rational number, [2]
   (b) a prime number. [1]

5. List all the possible integer values of x such that $2 \le x < 14$ and $15 \ge x > 8$. [3]

6. Solve the inequality $\frac{x-2}{4} < \frac{3x+1}{5} \le \frac{15-2x}{5}$ and illustrate your solution with a number line. [4]

7. Given that x is an integer such that $x + 3 < 15 < 4x - 3$, find the largest and smallest possible values of x. [3]

8. Solve the inequality $5(2x - 3) \geq 14 - x$ and state the smallest possible value of x if x is an integer. [3]

9. Given that $\frac{3x}{4} - \frac{1}{8} \leq 3x - 9\frac{1}{2}$, state the smallest value of x when
   (a) x is an integer, [2]
   (b) x is a prime number. [1]

10. Solve the inequality $\frac{x-3}{4} - \frac{x-5}{6} < \frac{2}{3}$. [3]

11. Solve the inequality $\frac{2}{3}(x-7) > 6 - 2x$ and show your answer on a number line. [3]

12. (a) Find the smallest integer x such that $-\frac{1}{4}x < 3$. [2]
    (b) Find the largest prime number y such that $5y \leq 45 + 2y$. [2]

13. Solve the inequality $\frac{x-3}{4} - \frac{x-5}{7} > 4$. [3]

14. Solve the inequality $\frac{2}{3}(x+2) \geq \frac{5}{6}$ and illustrate your answer on a number line. [3]

15. Given that $3x + 2 \leq 24$, solve the inequality and state
    (a) the greatest integer value of x,
    (b) the greatest prime number x. [4]

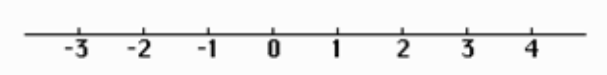
16. Find the largest prime number k for which $3k + 2 < 95$. [2]

17. Solve the following inequality: $5(x + 2) < 3(x-1) + x$. [2]

18. Solve the following inequalities, illustrating each solution with a number line.
    (a) $2x + 9 \geq 5$ [2]
    (b) $2(3 + x) < 6x - 9$ [2]

19. Solve the following inequality: $9 - \frac{3}{2}x \geq 12$ [2]

20. (a) Solve the inequality $\frac{5x-2}{-3} > 2-3x$ and indicate your answer on the number line given below. [3]



(b) If x is a prime number, state the smallest possible value of x. [1]

21. Given that $5(8-3x) \le 1\frac{1}{2}$, find the smallest possible value of x if
(a) x is an integer, [2]
(b) x is an odd number, [1]
(c) x is a factor of 32. [2]

22. Given that $17 - 4x \le x - 11$, find
(a) the least possible value of x, [2]
(b) the smallest integer value of x. [1]

23. Solve the inequality $2x - 1\frac{1}{2} \ge 11\frac{1}{4} + 5x$ and write down the largest integer value of x. [3]

24. Given that $3(x + 4) \ge 7(x - 1) - 2(x + 1)$, state the greatest possible value of x if x is
(a) a rational number, [2]
(b) an integer, [1]
(c) a prime number. [1]

25. Solve the inequality $5(2x - 3) > 6(3x - 1)$.
State the largest possible value of x if x is an integer. [4]

26. Solve the following inequality and illustrate your answer with a number line

$$\frac{2x+3}{3} < \frac{4x-9}{5}.$$ [3]

27. List all the possible values of x, where x is a prime number and satisfies both of the following inequalities: $2x > 19$, $3x + 2 < 81$. [3]

28. Solve the following inequalities and illustrate your answers with a number line respectively.
(a) $5 - 2x \geq 3x + 14$ [2]
(b) $-\frac{1}{3}(2x-3) \leq \frac{1}{3}(x+7)$ [3]

29. Solve the inequality $\frac{2-3x}{2} \geq 3\frac{1}{2} - 2x$ and draw a number line to illustrate your answer.
If x is an integer, state the smallest possible value of x. [4]

30. Given that $3x + 5 \leq \frac{1}{3}(2x + 48)$, find
(a) the greatest rational value of x, [2]
(b) the greatest value of x, if x is a prime number. [1]

31. Given that $\frac{2x+5}{3} \geq \frac{3x+2}{4} + \frac{4x-3}{3}$, solve the inequality and state
(a) the greatest rational number x, [2]
(b) the greatest value of x if x is a perfect square. [2]

32. Solve the following inequalities:
(a) $5 - 3x \leq 4x + 12$ [2]
(b) $\frac{3}{5} - \frac{2}{3}x > 1\frac{1}{4}x + \frac{1}{6}$ [3]

33. Given that $-5 \leq x \leq -1$ and $1 \leq y \leq 4$, find
(a) the greatest possible value of $2x - y$, [1]
(b) the least possible value of $\frac{2x}{y}$. [2]

34. Given that x and y are integers and $1 \leq x \leq 6$ and $-5 \leq y \leq 4$, find
(a) the greatest possible values of (i) $x - y$ (ii) $x^2 - y^2$ [2]
(b) the least possible values of (i) $x + y$ (ii) $\frac{y}{x}$ [3]

35. x and y are integers such that $-5 \leq x < 4$ and $-5 \leq y \leq 5$. Calculate
(a) the greatest value of $2x - y$, [1]
(b) the least value of $2xy$, [1]
(c) the greatest value of $x^2 + y^2$, [1]
(d) the least value of $2x^2 - y^2$. [1]

36. If $0.5 \leq x \leq 5$ and $-2 \leq y \leq 2$, find the greatest and least values of

(a) $2x - y$ (b) $\frac{y}{x}$ [4]

37. Two sides of a triangle are 10 cm and 6 cm and the third side has a length of x cm. Write down an inequality that must be satisfied by x. [2]

38. A fruit-seller bought a case of 113 oranges for $22.50. If he sells each orange for 40 cents, what is the least number of oranges that he must sell in order to make a profit of not less than $6? [3]

39. The perimeter of an equilateral triangle is not more than 90 cm. What is the largest possible side of the triangle? [2]

40. Yusof and his brother wanted to buy a present for their father. Yusof volunteered to pay $5 more than his brother. If the cost of the present was not more than $24, what was the greatest possible amount paid by Yusof? [4]

41. Mengli wants to buy hamburgers for her friends. Each hamburger costs $1.30. What is the maximum number of hamburgers she can buy with $22 and what will be the change received? [4]

42. Mani and Usha went shopping. During their shopping spree, Usha spent $25 more than Mani. Together they spent at least $120. What is the least amount spent by Usha? [4]

43. Find the odd integer which satisfies the inequalities $2x + 1 \geq 5$ and $3x + 15 > 5x - 1$. [2]

44. Given that $-2 \leq x \leq 3$ and $-3 \leq y \leq -1$, calculate

(a) the smallest value of $x - y$, [1]

(b) the largest value of $\frac{x}{y}$. [2]

45. Given that $-5 \leq 4x + 1 \leq 2x + 9$ and $-6 \leq 2y - 2 \leq 8$, find

(a) the greatest value of $x - y$, [1]

(b) the smallest value of $(x + y)(x - y)$. [2]

46. Given that x is a rational number and that $\frac{1}{2} \leq x \leq 39\frac{1}{4}$, write down

(a) the greatest value of x,

(b) the smallest value of x such that x is a prime number,

(c) the greatest integer value of x which is exactly divisible by 2 and 5. [3]

47. Solve the inequality $\frac{x+3}{2} > 2$. [2]

48. Solve the inequalities

(a) $\frac{2x}{3} - \frac{x}{2} \geq \frac{5}{6}$ [2]

(b) $\frac{x+1}{2} - \frac{x+3}{4} \geq \frac{3x-5}{8}$ [2]

49. If $12 - 7x \leq 5 - 2x$, find the least possible value of x. [2]

50. Solve the following inequalities and illustrate your answer on a number line.

(a) $\frac{1}{3}(x + 2) \leq 3x + 2$ [2]

(b) $7 + 3x < 5 - x \leq 6 - 3x$ [3]

51. Given that $-2 \leq x \leq 3\frac{1}{2}$ and $2 \leq y \leq 5$,

(a) list the integer values of x, [1]

(b) write down the largest rational value of x, [1]

(c) calculate the smallest possible value of

(i) $(x - y)^2$ [1]

(ii) $x^2 - y^2$ [1]

(iii) $\frac{2x}{y}$ [1]

52. Given that $-5 \leq 4x - 1 \leq 2x + 7$ and $-6 \leq 3y \leq 15$, find

(a) the greatest possible value of $x + y$

(b) the smallest possible value of $x - y$

(c) the greatest possible value of $x^2 - y^2$

(d) the smallest possible value of $x^2 + y^2$ [6]

53. A woman buys x oranges at 50 cents each and $(2x + 1)$ pineapples at \$1.20 each. If she wishes to spend not more than \$25 on these produce,

(a) form an inequality in x, and [2]

(b) find the largest number of x. [1]

54. Given that $1 \leq x \leq 8$ and $-5 \leq y \leq 1$, find

(a) the greatest possible value of $x - y$ [1]

(b) the smallest possible value of $x^2 + y^2$ [2]

55. Given that $3x \leq 42\frac{1}{2}$, state the largest possible value of x if

(a) x is an integer, [1]

(b) x is a prime number, [1]

(c) x is a real number. [1]

56. Find the smallest integer value of x that satisfies the inequality $2x - 3(1 - x) > 7$. [3]

57. Given that $3 \le x \le 5$ and $-1 \le y \le 3$, find

(a) the largest value of $3x - y$, [1]

(b) the smallest value of $\frac{1}{x} + \frac{1}{y}$. [2]

58. Find the integer values of x for which $21 < 3(x + 1) < 30$. [2]

59. Solve the inequality $\frac{3}{4}x - \frac{2}{3}(1 - x) < 7$. [2]

60. Find the possible values of x for which x is a positive integer and $3.5 < \frac{22}{7}x^2 < 143$. [3]

61. Solve each of the following inequalities, illustrating your answer with the number line. [36]

(a) $2x - 3 > 4$ (b) $3x + 4 < 7$ (c) $7x - 12 < 9$

(d) $4x + 1 > -3$ (e) $3x + 2 \ge 11$ (f) $5x - 4 \le 21$

(g) $3x + 24 \ge 7x$ (h) $5x - 12 \ge 2x$ (i) $8x - 4 \ge 3x + 16$

(j) $7x - 13 > 3x - 5$ (k) $6x - 9 \le 2x - 7$ (l) $15 - 3x < x + 4$

(m) $\frac{x+1}{10} < \frac{2x-7}{15}$ (n) $\frac{x-3}{21} > \frac{x-7}{14}$ (o) $\frac{x}{11} + \frac{13}{44} \le \frac{x}{5} - \frac{1}{11}$

(p) $\frac{2x}{35} + \frac{6}{7} > \frac{x}{5} - \frac{2}{5}$ (q) $\frac{1}{2} + \frac{x-1}{2} > \frac{2x}{7} + \frac{1}{14}$ (r) $\frac{x}{3} - \frac{1}{5} > \frac{13}{30} - \frac{13-5x}{10}$

62. Given that $x \ge 9\frac{1}{2}$, state the smallest possible value of x if

(a) x is a prime number,

(b) x is a mixed number,

(c) x is an integer. [3]

63. Given that $4x - 3 \le 18$, find the greatest possible value of x if

(a) x is an integer,

(b) x is a rational number,

(c) x is a prime number. [4]

64. Find (a) the smallest integer x such that $7x > 18$,

(b) the largest prime number x such that $\frac{3x}{4} < 18$,

(c) the smallest mixed number x such that $\frac{2}{5}x \ge 13$,

(d) the largest rational number such that $\frac{1}{3}x - 5 \le 14 - \frac{2}{5}x$. [5]

65. An apple costs 45 cents while oranges are 35 cents each. A man wishes to buy 27 apples and 46 oranges. What is the minimum number of $10 notes he must bring to make the purchase? [3]

66. A woman is organising a barbecue party for her friends. She intends to buy 8 kg of beef costing $ 9.80 per kg, 12 kg of mutton costing $12.50 per kg, 16 kg of chicken wings costing $4.20 per kg and 17 kg of prawns at $15.50 per kg. What is the minimum number of $50 notes she must bring along for all these purchases? [4]

67. A music shop is having a sale and each compact disc is priced at $12.49. A man has $97 in his pocket. What is the maximum number of compact discs that he can buy? [3]

68. Solve the following inequalities. [16]

(a) $x + 5 < 5x - 9$ (b) $2(3x - 1) \leq (4 - x)$

(c) $\frac{2x}{3} - \frac{x}{2} \leq \frac{4}{5}$ (d) $\frac{x+3}{2} - \frac{x-1}{4} \leq \frac{3x+8}{8}$

(e) $x + 17 < 3(x + 5) < 45$ (f) $3x - 10 > 4x - 19 > x + 2$

(g) $4x - 4 > 3x > 4x - 6$ (h) $\frac{x-1}{3} < \frac{2x+2}{5} \leq 4$

69. Given that $-5 \leq x \leq -1$ and $1 \leq y \leq 6$, find

(a) the greatest possible value of $2x - y$, (b) the greatest possible value of $y - 4x$,

(c) the least possible value of $\frac{y}{x}$, (d) the least possible value of $\frac{x}{y}$.

[8]

70. List the integer values of x, where x is prime, which satisfy both the following inequalities

$$2x > 14,\ 3x - 2 < 67.$$ [3]

71. List the integer values of x which satisfy $3x - 5 < 26 \leq 4x - 5$. [3]

72. Find the integer x for which $3 < x - 3 < 7$ and $11 < 2x + 3 < 20$. [3]

73. Solve the inequality $3x + 5 \leq 4x + 1 \leq 3x + 8$. [3]

74. Given that $-7 < 2x \leq 8$, write down

(i) the greatest integer value of x, (ii) the smallest integer value of x. [4]

Answers

1. (a) –3, 4 (b) –6, 8 (c) –5, 3 (d) –6, 2

2. (a) –2 (b) 7 (c) $8\frac{2}{3}$

4. (a) $3\frac{1}{10}$ (b) 7

5. 9, 10, 11, 12, 13

6. $-2 < x \leq 2\frac{4}{5}$

7. Largest possible value of $x = 11$
   Smallest possible value of $x = 5$

8. $x \geq 2\frac{7}{11}$, 3

9. (a) 5 (b) 5

10. $x < 7$

11. $x > 4$

12. (a) 0 (b) 13

13. $x > 37\frac{2}{3}$

14. $x \geq -\frac{3}{4}$

15. (a) 7 (b) 7

16. 29

17. $x < -13$

18. (a) $x \geq -2$ (b) $x > 3\frac{3}{4}$

19. $x \leq -2$

20. (a) $x > 1$ (b) 2

21. (a) 3 (b) 3 (c) 4

22. (a) $5\frac{3}{5}$ (b) 6

23. $x \leq 4\frac{1}{4}$; 4

24. (a) $10\frac{1}{2}$ (b) 10 (c) 7

25. $x < -1\frac{1}{8}$, -2

26. $x > 21$

27. 11, 13, 17, 19, 23

28. (a) $x \leq -1\frac{4}{5}$ (b) $x \geq -1\frac{1}{3}$

29. $x \geq 5$; 5

30. (a) 9 (b) 7

31. (a) $1\frac{9}{17}$ (b) 1

32. (a) $x \geq -1$ (b) $x < \frac{26}{115}$

33. (a) –3 (b) –10

34. (a)(i) 11 (ii) 36
(b)(i) –4 (ii) –5

35. (a) 13 (b) –50 (c) 50 (d) 50

36. (a) 12, –1 (b) 4, –4

37. $4 < x < 16$

38. 72

39. 30 cm

40. $14.50

41. 16 ; $1.20

42. $72.50

43. 3, 5, 7

44. (a) -1 (b) 2

45. (a) 6 (b) -25

46. (a) $39\frac{1}{4}$ (b) 2 (c) 30

47. $x > 1$

48. (a) $x \geq 5$ (b) $x \leq 3$

49. $1\frac{2}{5}$

50. (a) $x \geq -\frac{1}{2}$ (b) $x \leq -\frac{1}{4}$

51. (a) $-2,\ -1,\ 0,\ 1,\ 2,\ 3$ (b) $3\frac{1}{2}$ (c) (i) 0 (ii) -25 (iii) -2

52. (a) 9 (b) -6 (c) 16 (d) 0

53. (a) $2.9x \leq 23.8$ (b) 8

54. (a) 13 (b) 1

55. (a) 14 (b) 13 (c) $14\frac{1}{6}$

56. 3

57. (a) 16 (b) $-\frac{4}{5}$

58. 7, 8

59. $x < 5\frac{7}{17}$

60. 2, 3, 4, 5, 6

61 (a) $x > 3\frac{1}{2}$ (b) $x<1$ (c) $x<3$ (d) $x>-1$ (e) $x \geq 3$
(f) $x \leq 5$ (g) $x \leq 6$ (h) $x \geq 4$ (I) $x \geq 4$ (j) $x>2$
(k) $x \leq \frac{1}{2}$ (l) $x > 2\frac{3}{4}$ (m) $x>17$ (n) $x<15$ (o) $x \geq 3\frac{13}{24}$
(p) $x < 8\frac{4}{5}$ (q) $x > \frac{1}{3}$ (r) $x<4$

62. (a) 11 (b) $9\frac{1}{2}$ (c) 10

63. (a) 5 (b) $5\frac{1}{4}$ (c) 5

64. (a) 3 (b) 23 (c) $32\frac{1}{2}$ (d) $25\frac{10}{11}$

65. 3

66. 12

67. 7

68. (a) $x > 3\frac{1}{2}$ (b) $x \leq 1$ (c) $x \leq 4\frac{4}{5}$ (d) $x \geq 6$
(e) $1 < x < 10$ (f) $7 < x < 9$ (g) $4 < x < 6$ (h) $-11 < x \leq 9$

69. (a) –3 (b) 26 (c) –6 (d) –5

70. 11, 13, 17, 19

71. 7, 8, 9, 10

72. 7, 8

73. $4 \leq x \leq 7$

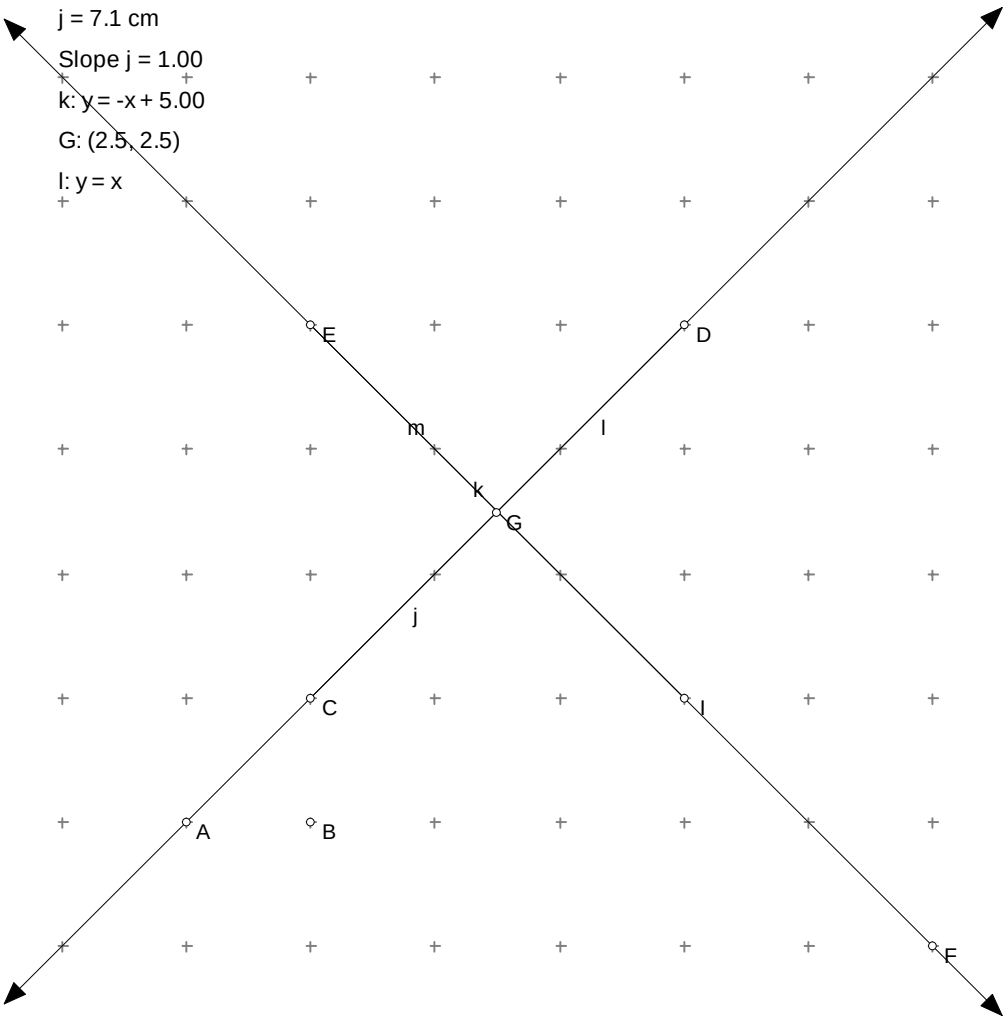
74. (i) 4 (ii) – 4

Chapter 4

Secondary 3 Mathematics
Chapter 4 Coordinate Geometry

IT Activities
Geometer's Sketchpad

WS 3-4-1 using GSP - Page 83



1. Select Create Axes and then Show Grid from the Graph Menu. A Grid showing two points A at (0, 0) and B at (1, 0) will be shown.

2. Select  and drag the point A if you want to change the position of the axes. Dragging the point B will change the scale of the axes.

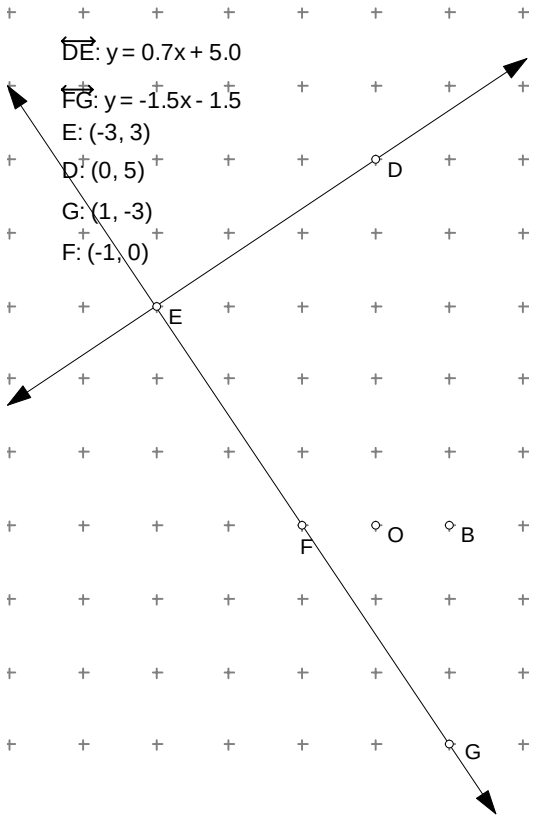
3. Use  to draw a line segment joining the points C(1, 1) and D(5, 2). Using  and select line segment CD, you can measure the Length and the Slope of the line segment from the Measure Menu.

4. To find the equation of the line you need to use  to draw line(one with double arrow) and select Equation from Measure Menu. Use  to draw a line starting from the point (1. 3) and ending at the point (6, 1).

5. You can use  to drag the point E and observe the change in the displayed equation. Do the same for the point F.

6. To find the point of intersection of the two lines, use  to mark the point as G and choose Coordinates from Measure Menu.

7. Use  to drag point C and observe the change in the coordinates of G.



1. Use Graph Menu to select Create axes and then Show grid

2. Re-label the origin as O with  . Double click on point and a Re-label panel will appear. Type O at the blank and click OK.

3. Use  to select 2 points and label them as D and E

4. Use  (double arrow head) to draw a line passing through the points D and E

5. Use  to select line DE and choose Equation from the Measure Menu.

6. Use  to select D and E and choose Co-ordinates from Measure Menu

7. Choose two more points by using  and label them as F and G. Draw the line through F and G.

8. Use  to select line FG and Equation from Measure Menu to find its equation.

9. Use  to drag the point D and see how the equation changes.

10. Repeat the dragging with other points and see the effect for yourself.

Secondary 3 Mathematics
Chapter 4 Coordinate Geometry

GENERAL NOTES

Teachers should revise with pupils the naming of coordinate points on a graph. It is not uncommon to find even Secondary 4 pupils being confused by the naming of coordinate points.

Many pupils have difficulty distinguishing equations parallel to the x- and y- axes. Many incorrectly refer to the equation of the x-axis as $x = 0$ and the equation of the y-axis as $y = 0$. Constant revision over the year may help to correct this common mistake.

To facilitate easy understanding of a problem in coordinate geometry, teachers may insist that pupils draw a sketch of the points for every question they are attempting.

The subtopic on mid-point of two end points is moved to the additional mathematics syllabus. Although the concept of parallel lines implying that the gradients are the same is not mentioned in the syllabus, it is still in the additional mathematics syllabus. Teachers can mention this fact to the pupils as many pupils nowadays take additional mathematics.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: 35 min

Class: _____ Marks: 9

Secondary 3 Multiple-Choice Questions

Chapter 4 Coordinate Geometry

1. Find the equation of the straight line that passes through the points $A(0, -3)$ and $B(-1, 0)$.

   (A) $y = 3x - 3$ (B) $y = -3x - 3$ (C) $y = -\frac{1}{3}x - 3$

   (D) $y = \frac{1}{3}x - 3$ (E) $y = -x - 3$ ( )

2. A straight line has a gradient of 2 and passes through the point (3, 4). The equation of the straight line is

   (A) $2x - y - 2 = 0$ (B) $2x - y + 2 = 0$ (C) $x - 2y - 2 = 0$

   (D) $x - 2y + 2 = 0$ (E) $2x - y - 10 = 0$ ( )

3. Which of the following lines is parallel to the line $4y = 6x + 5$?

   (A) $2y + 3x = 5$ (B) $2y - 3x = 57$ (C) $y = 6x + 5$

   (D) $4y + 6x = 13$ (E) none of the above ( )

4. The length between points (2, –5) and (–1, –2) is

   (A) $3\sqrt{2}$ (B) $\sqrt{10}$ (C) $5\sqrt{2}$ (D) $\sqrt{58}$ (E) $2\sqrt{13}$ ( )

5. The equation $x = 15$ represents a straight line

   (A) parallel to the x-axis. (B) parallel to the y-axis.

   (C) passing through the origin. (D) having a gradient of 15.

   (E) having a y-intercept of 15. ( )

6. The equation $y + 15 = 0$ represents a straight line

   (A) parallel to the x-axis. (B) parallel to the y-axis.

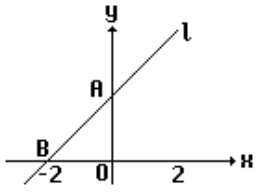
   (C) passing through the origin. (D) having a gradient of 15.

   (E) having a gradient of –15. ( )

7. In the diagram, the line l has the equation $2y = mx + 2c$. The length of OA is

   (A) 2 units (B) 1 unit (C) $2c$ units

   (D) c units (E) $\frac{m}{2}$ units



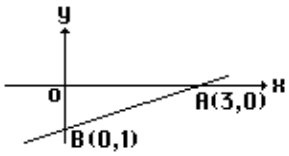
( )

8. The distance between the points $P(a, 5)$ and $Q(-3, 2)$ is 5 units. The value of a must be

(A) 1 (B) –7 (C) 7 (D) 1 or –7 (E) 7 or –1 ( )

9. The equation of the line AB in the diagram is

(A) $x + 3y - 3 = 0$ (B) $x - 3y - 3 = 0$
(C) $3x + y - 3 = 0$ (D) $3x - y - 3 = 0$
(E) $x - 3y + 3 = 0$



( )

Answers

1. B	**2.** A	**3.** B	**4.** A	**5.** B
6. A	**7.** D	**8.** D	**9.** B	

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: 

Secondary 3 Mathematics Test
Chapter 4 Coordinate Geometry

1. If (3, 7) is a point on the line $kx + 3y = 37$, find the value of k. [2]
Using separate diagrams, sketch the graphs of
(a) $y = kx^2$
(b) $y = \frac{k}{x}$, where k is a positive constant and x is not equal to zero in (b). [4]

2. A straight line passes through the points (0, 5) and (2, 13). Find
(a) its gradient, [1]
(b) its equation. [2]

3. The coordinates of P, Q and R are $P(8, 12)$, $Q(9, 4)$ and $R(-5, -4)$ respectively. M is the mid-point of QR. Calculate the distance of PM. [4]

4. Find the equation of the straight line
(a) passing through the point (5, 7) and parallel to the x-axis. [1]
(b) passing through the point (–2, –5) and parallel to the y-axis. [1]

5. Find the equation of the straight line l which passes through the points (0, 1) and (–2, 4). If the point (k, 9) lies on l, find the value of k. [3]

6. A straight line with gradient m and y-intercept c passes through the points (2, 5) and (– 4, 9). Find the values of m and c. [3]

7. The points (3, –2), (–2, 5) and (5, k) lie on a straight line. Find the value of k. [2]

8. The equation of the line l is $\frac{x}{3} + \frac{y}{4} = 1$.
(a) Find its gradient. [1]
(b) The line cuts the x-axis at A and the y-axis at B. Find the area of ΔOAB where O is the origin. [2]

9. Find the equation of the straight line which passes through the point (–2, 4) and is parallel to the line $y = 2x + 5$. [3]

10. Find the equation of the straight line which has a gradient of 2 and which passes through the point (2, 1). [3]

11. **(a)** Given that the line $2x + 3y = k$ passes through the point $(2, -4)$, find k. [1]
(b) The line $5x + 7y = 15$ is parallel to the line $2y = kx + 13$. Find the value of k. [2]

12. The point $(-2, t)$ lies on the line $3y + 2x = 7$. Find the value of t. [2]

13. The equation of a straight line is $2x + y = 8$.
(a) Find the gradient of the line. [1]
(b) Given that the point $(-3, k)$ lies on the line, find the value of k. [2]

14. The coordinates of the three points are $A(0, 3)$, $B(t, 0)$ and $C(\frac{1}{2}t, t)$. If $AB = 2AC$, find the possible values of t. [3]

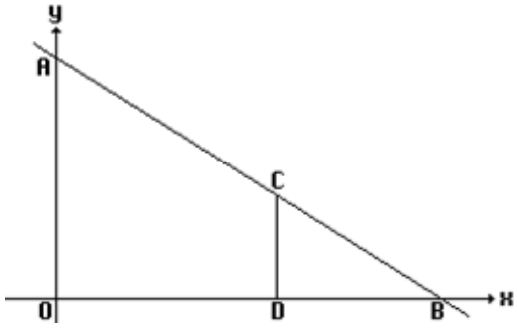
15. Given that the three points $A(0, 1)$, $B(k, 2.5)$ and $C(2k, 4)$ lie on a straight line, find the value of k. [3]

16. **(a)** Find the gradient of the line $\frac{x}{3} + \frac{y}{5} = 1$. [2]
(b) The straight line $y = mx + c$ is parallel to the line $3x + 2y = 13$ and passes through the point $(1, 1\frac{1}{2})$. Find the values of m and c. [3]

17. The points $(2, 0)$ and $(-2, 7)$ lie on the line $kx + ky + 5 = 0$. Find the values of h and k. [3]

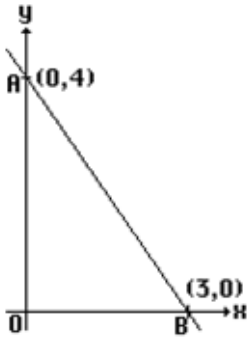
18. Write down
(a) the gradient of the straight line $3x + 5y = 17$. [1]
(b) the coordinates of the point on the line $4x - 5y = 13$ which has $y = -1$ as its y coordinate. [1]

19. In the diagram, the coordinates A and C are $(0, 6)$ and $(5, 2)$ respectively. The line AC produced cuts the x-axis at B, and D is a point on the x-axis where CD is parallel to the y-axis.



(a) Find the gradient of the line AC. [1]
(b) Find the equation of the line AB. [1]
(c) Find the coordinates of B. [1]
(d) Calculate the area of ΔBCD. [2]

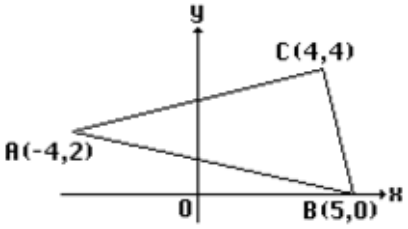
20. The diagram shows a line segment AB where A is the point (0, 4) and B is the point (3, 0).



(a) Find the equation of the line AB. [2]
(b) If the line AB is reflected in the y-axis, find the equation of the image of the line. [2]

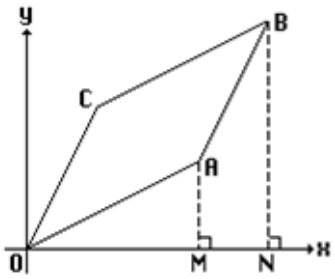
21. **(a)** If the straight line $3y = k - 2x$ passes through $(-1, -5)$, find k. [1]
(b) If the gradient of the straight line $(2k - 1)y + (k + 1)x = 3$ is parallel to the line $y = 3x - 7$, find the value of k. [2]
(c) Find the equation of the line joining the points $A(1, 5)$ and $B(7, 2)$. [2]

22. The coordinates of ΔABC are $A(-4, 2)$, $B(5, 0)$ and $C(4, 4)$.



(a) Calculate the lengths of AB, BC and AC. [3]
(b) Show that ΔABC is a right-angled triangle. [2]
(c) Calculate the area of ΔABC. [2]
(d) Calculate the perpendicular length from C to AB. [2]

23. The coordinates of the points O, A and B of the parallelogram $OABC$ are (0, 0), (6, 3) and (10, 8) respectively. Calculate



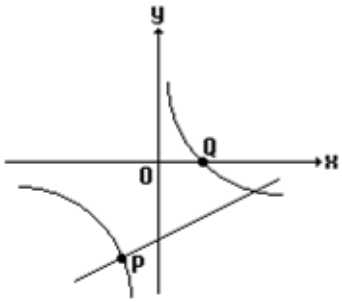
(a) the mid-point of OB. [1]
(b) the coordinates of C. [2]
(c) the areas of ΔOAM, ΔOBN and the trapezium $ABNM$. [4]
(d) the area of the parallelogram $OABC$. [2]

24. The equation of a straight line is $2x + 5y = 20$. Find
(a) the gradient of the line, [1]
(b) the coordinates of the point where the line crosses the y-axis, [1]
(c) the coordinates of the point at which the line intersects the line $x = 2\frac{1}{2}$, [1]
(d) the equation of the line which is parallel to $2x + 5y = 20$ and which passes through the point $(-1, 7)$. [2]

25. l is the line passing through the point (3, 8) and parallel to the line $2x - 3y + 5 = 0$. Find the equation of the line l. [3]

26. **(a)** Given that the points $A(-2, 3)$, $B(2, -2)$ and $C(6, 1)$ are three vertices of the parallelogram $ABCD$, find the coordinates of the point D. [3]
(b) Given also that the point E has coordinates $E(5, k)$ and that A, B and E are collinear, find the value of k. [2]
(c) Find the equation of the line passing through C and parallel to the line $5x - 7y = 84$. [3]
(d) Given that the area of ΔABC is 16 units^2, calculate the perpendicular distance from B to AC, giving your answer correct to 2 decimal places. [4]

27. The diagram shows part of the graph of $y = \frac{2}{x} - 3$.



(a) A point P is on the curve with coordinates $(-1, k)$. Find the value of k. [1]
(b) Given that the straight line $y = 2x + h$ passes through P, find the value of h. [1]
(c) Another straight line l, parallel to $y = 2x + h$, passes through the point Q. Find the equation of the line l. [2]

28. The gradient of the line joining the points $(5, k)$ and $(k, -3)$ is $\frac{2}{3}$. Calculate the value of k. [3]

29. Given that the coordinates of A and B are (1, 3) and (7, 1) respectively, find
(a) the gradient of AB (b) the equation of AB. [4]

30. A line passes through the points $(-2, k)$ and (5, 9). If the gradient of the line is $\frac{4}{7}$, find the value of k. This line cuts the x-axis at P and the y-axis at Q. Find the area of ΔOPQ, where O is the origin. [3]

31. (a) A straight line passes through $P(1, -1)$, $Q(4, 1)$ and $R(k, 3)$. Find the value of k.
(b) Find the equation of a straight line passing through the points $A(4, 3)$ and $B(0, -5)$. [4]

32. (a) Find the equation of a line which has gradient $\frac{2}{3}$ and which passes through the point
$(-2, 5)$. Give your answer for the equation in the form $y = mx + c$.
(b) Find the equation of the line on which both the points (0, 3) and (2, 5) lie. [6]

33. The straight line $y = x + 2$ cuts the x-axis at point A and the y-axis at the point B. Calculate the area of ΔAOB where O is the origin. [3]

34. Write down the gradient of the straight line $2x + 3y = 4$ and find the equation of the line which is parallel to $2x + 3y = 4$ and which passes through the point (1, 7). [4]

35. For all values of m, the line $2y = mx + 6$ passes through a fixed point K. State the coordinates of K. [2]

36. Find the equation of the line parallel to $3x + 4y + 15 = 0$ and passing through the point (3, 7). [3]

37. The straight line $3y = mx + c$ is parallel to the line $2y - 3x = 5$ and passes through the point (1, 12). Find the value of m and of c. [3]

38. Given that line $\frac{x}{4} - \frac{y}{6} = 1$, find

(a) its gradient, (b) the coordinates of the point at which it cuts the line $x = 8$. [4]

39. The lines $ky - 2x + 5 = 0$ and $6y - (k + 1)x - 3 = 0$ are parallel. Find the value(s) of k. [2]

40. Three of the vertices of a parallelogram $ABCD$ are $A(-1, 5)$, $B(5, 1)$ and $C(6, -2)$. Find the coordinates of the fourth vertex D. [3]

41. The curve $x^2 + y^2 - 6x - 8y = 0$ cuts the x-axis at points O and A and the y-axis at O and B.

(a) Find the coordinates of O, A and B.
(b) Find the gradient of the line AB.
(c) Write down the equation of the line AB.
(d) ΔOAB is rotated through 90° clockwise about O to ΔOPQ. Write down the coordinates of P and Q. [8]

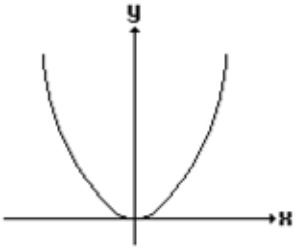
42. P, Q and R are the points (8, 3), (6, k) and (–3, –8) respectively.

(a) If P, Q and R are collinear, find k.
(b) With this value of k, find the ratio $PQ : QR$.
(c) RQ is produced to T so that $RQ = 2QT$. Find the coordinates of T. [7]

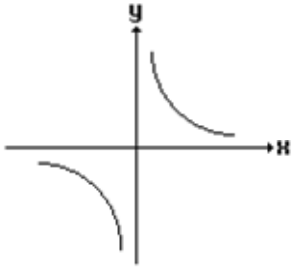
Answers

1.

(a)



(b)



2. (a) 4 (b) $y = 4x + 5$

3. $\sqrt{180} = 13.4$

4. (a) $y = 7$ (b) $x = -2$

5. $2y + 3x = 2$; $k = -5\frac{1}{3}$

6. $m = -\frac{2}{3}$, $c = 6\frac{1}{3}$

7. $k = -4\frac{4}{5}$

8. (a) $-1\frac{1}{3}$ (b) 6

9. $y = 2x + 8$

10. $y = 2x - 3$

11. (a) $k = -8$ (b) $k = -1\frac{3}{7}$

12. $t = 3\frac{2}{3}$

13. (a) -2 (b) $k = 14$

14. $t = 4\frac{1}{2}$ or $1\frac{1}{2}$

15. $k = 3$

16. (a) $-1\frac{2}{3}$ (b) $m = -1\frac{1}{2}$; $c = 3$

17. $h = -2\frac{1}{2}$, $k = -1\frac{3}{7}$

18. (a) $-\frac{3}{5}$ (b) (2, –1)

19. (a) $-\frac{4}{5}$ (b) $5y + 4x = 30$
(c) (7.5, 0) (d) 2.5 units^2

20. (a) $3y + 4x = 12$ (b) $3y = 4x + 12$

21. (a) –17 (b) $\frac{2}{7}$ (c) $2y + x = 11$

22. (a) $\sqrt{85}$, $\sqrt{17}$, $\sqrt{68}$ (c) 17 (d) 3.69

23. (a) (5, 4) (b) (4, 5) (c) 9, 40, 22 (d) 18

24. (a) $-\frac{2}{5}$ (b) (0, 4)
(c) $(2\frac{1}{2}, 3)$ (d) $2x + 5y = 33$

25. $3y = 2x + 5$

26. (a) $D(2, 6)$ (b) $k = -5\frac{3}{4}$ (c) $7y = 5x - 23$ (d) 3.88 units

27. (a) –5 (b) –3 (c) $3y = 6x - 4$

28. $\frac{1}{5}$ **29.** (a) $-\frac{1}{3}$ (b) $3y + x = 10$

30. $k = 5, 33\frac{1}{56}$ **31.** (a) 7 (b) $y = 7x - 5$

32. (a) $y = \frac{2}{3}x + 6\frac{1}{3}$ (b) $y = x + 3$ **33.** 2 units^2

34. $-\frac{2}{3}$, $3y + 2x = 23$ **35.** (0, 3) **36.** $4y + 3x + 37 = 0$

37. $m = 1\frac{1}{2}$, $c = 9$ **38.** (a) $1\frac{1}{2}$ (b) (8, 6)

39. 3, – 4 **40.** (0, 2)

41. (a) (0, 0) , (6, 0) , (0, 8) (b) $-1\frac{1}{3}$

(c) $3y + 4x = 24$ (d) $P\,(0, -6)$, $Q\,(8, 0)$

42. (a) 1 (b) 2 : 9 (c) $(10\frac{1}{2}\ , 5\frac{1}{2})$

Chapter 5

Secondary 3 Mathematics
Chapter 5 Matrices

GENERAL NOTES

In this syllabus, only addition, subtraction, multiplication of two matrices will be involved. The subtopic of using inverse matrix to solve a pair of simultaneous equations will be covered in the additional mathematics syllabus. The topic of using matrices to solve practical problems is brought from the additional mathematics syllabus to the 'O' level mathematics syllabus. Pupils are normally weak at recognising the order of a matrix and this normally leads to the error of multiplying two incompatible matrices for matrix multiplication. It is worth to emphasise the importance of first determine whether two matrices may be multiplied.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: 

Secondary 3 Mathematics Test
Chapter 5 Matrices

1. Given that $\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{B} = (-1, 4)$, find

(a) $\mathbf{AB}$, **(b)** $2\mathbf{BA}$. [4]

2. Given that $\mathbf{A} = \begin{pmatrix} -6 & -16 \\ 13 & 9 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -1 & -5 \\ 4 & 6 \end{pmatrix}$, find the matrices $\mathbf{X}$ and $\mathbf{Y}$ such that

(a) $3\mathbf{A} - 2\mathbf{X} = \mathbf{B}$, **(b)** $\mathbf{YB} = \mathbf{A}$. [5]

3. Given that $(1 \;\; 3)\begin{pmatrix} x \\ 3 \end{pmatrix} = (1 \;\; 3 \;\; -1)\begin{pmatrix} x \\ 2x \\ 3 \end{pmatrix}$, find the value of x. [3]

4. Given that $\begin{pmatrix} 3 & 4 & t \\ -2 & 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ s \end{pmatrix}$, find the value of s and of t. [4]

5. Given that $\begin{pmatrix} 3 & 0 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & 4 \end{pmatrix} = \begin{pmatrix} x \\ 4 \end{pmatrix}(2 \;\; 1)$, find the values of a, b, c and x. [4]

6. Given that $\begin{pmatrix} 2x & 0 \\ 3y & -z \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \end{pmatrix}$ and $\begin{pmatrix} 2x & 0 \\ 3y & -z \end{pmatrix}\begin{pmatrix} 5 \\ -15 \end{pmatrix} = \begin{pmatrix} 10 \\ -15 \end{pmatrix}$, find the values of x, y and z. [3]

7. Given that $\begin{pmatrix} 2 & 0 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 16 \end{pmatrix}$, find the values of $4x - y$.

8. Given that $\begin{pmatrix} a & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} -3 \\ b \end{pmatrix} = \begin{pmatrix} 7 \\ 15 \end{pmatrix}$, find the value of a and of b. [2]

9. Given that $\begin{pmatrix} 3 & 0 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} k & 0 \\ 3 & 3h \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ m & 2h-7 \end{pmatrix}$, find the values of h, k *and* m. [3]

10. Given that $\mathbf{A} = \begin{pmatrix} -1 & -1 \\ 3 & 3 \end{pmatrix}$, find the matrices $\mathbf{A}^2$ and $\mathbf{A}^3$. Hence, write down the matrices $\mathbf{A}^5$ and $\mathbf{A}^7$. [4]

11. Given that x and y are positive integers and that $\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 13$, find the possible values of x and y. [3]

12. Solve the following matrix equations.

(a) $\begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ a \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ b \end{pmatrix}$

(b) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} a & -4 \\ b & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -3 \\ 6 & 2c \end{pmatrix}$

(d) $3\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & -2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(e) $\begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 1 & 7 \\ 3 & \frac{3}{4} \end{pmatrix} - \begin{pmatrix} 5 & 7 \\ 3 & 7 \end{pmatrix}$ [10]

13. If $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$,

(a) is $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$?
(b) is $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$?
(c) is $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$?
(d) is $\mathbf{A} \times \mathbf{B} = \mathbf{B} \times \mathbf{A}$?
(e) is $\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{C})$? Can you give the name of this rule?
(f) is $\mathbf{A} + (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} + \mathbf{B}) \times (\mathbf{A} + \mathbf{C})$? [6]

14. If $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, work out each of the following and express your answers in terms of **A**, **B**, **C** and **D**.

(a) $\mathbf{A}^2$ (b) $\mathbf{B}^2$ (c) $\mathbf{C}^2$ (d) $\mathbf{D}^2$
(e) **DB** (f) **BD** (g) **BC** (h) **CB** [16]

15. Use the matrices $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$ to verify the distributive law of multiplication over addition, that is,

$\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$
$(\mathbf{B} + \mathbf{C})\mathbf{A} = \mathbf{BA} + \mathbf{CA}$ [6]

16. Solve the following matrix equations.

(a) $\begin{pmatrix}2 & 0\\ 2 & 1\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=\begin{pmatrix}6\\ 25\end{pmatrix}$ (b) $\begin{pmatrix}a & b\end{pmatrix}\begin{pmatrix}5 & 6\\ 0 & 3\end{pmatrix}=\begin{pmatrix}10 & 55\end{pmatrix}$ [4]

17. Find the unknowns in each of the following matrix equations:

(a) $\begin{pmatrix}p & q\end{pmatrix}\begin{pmatrix}3 & 0\\ -4 & 2\end{pmatrix}=\begin{pmatrix}-3 & 6\end{pmatrix}$ (b) $\begin{pmatrix}a & b\\ 3 & 2a\end{pmatrix}\begin{pmatrix}1\\ 4\end{pmatrix}=\begin{pmatrix}15\\ 11\end{pmatrix}$

(c) $\begin{pmatrix}x & 2\\ 2z & 0\end{pmatrix}\begin{pmatrix}4\\ -1\end{pmatrix}=\begin{pmatrix}-2\\ 8\end{pmatrix}$ (d) $\begin{pmatrix}3 & x\\ 5 & 0\end{pmatrix}\begin{pmatrix}x\\ x\end{pmatrix}=\begin{pmatrix}y\\ -10\end{pmatrix}$

(e) $\begin{pmatrix}x & 3\\ -1 & 4\end{pmatrix}\begin{pmatrix}4 & y\\ 1 & x\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ (f) $\begin{pmatrix}x & y\\ 8 & -4\end{pmatrix}\begin{pmatrix}x & y\\ 8 & -4\end{pmatrix}=\begin{pmatrix}x & y\\ 8 & -4\end{pmatrix}$ [12]

18. Find the value of each of the unknowns in the following.

(a) $\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}+3\begin{pmatrix}7 & 5\\ -1 & a\end{pmatrix}=\begin{pmatrix}b & c\\ d & 2a\end{pmatrix}$ (b) $\begin{pmatrix}1 & -1 & 3\\ 2 & -3 & 4\end{pmatrix}-\begin{pmatrix}a & b & c\\ 4 & 3 & 9\end{pmatrix}=\begin{pmatrix}6 & -7 & 3\\ h & k & t\end{pmatrix}$

(c) $\begin{pmatrix}2 & 1\\ -4 & -6\\ -3 & 8\end{pmatrix}+3\begin{pmatrix}x & y\\ -1 & 4\\ 9 & 2\end{pmatrix}=\begin{pmatrix}x & 6\\ h & 2k\\ 3t & 14\end{pmatrix}$ (d) $\begin{pmatrix}4 & a & b\\ -1 & a & 2\end{pmatrix}\begin{pmatrix}1\\ 2\\ -3\end{pmatrix}=\begin{pmatrix}-7\\ -7\end{pmatrix}$

(e) $\begin{pmatrix}7 & -2\\ -10 & 3\end{pmatrix}\begin{pmatrix}p & q\\ r & s\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ (f) $\begin{pmatrix}a & -2\\ -5 & 1\end{pmatrix}\begin{pmatrix}-1\\ b\end{pmatrix}=\begin{pmatrix}-11\\ 10\end{pmatrix}$

(g) $\begin{pmatrix}3 & -1\\ a & b\end{pmatrix}\begin{pmatrix}-2 & c\\ 4 & -3\end{pmatrix}=\begin{pmatrix}-2b & 6\\ 22 & d\end{pmatrix}$ (h) $\begin{pmatrix}2 & 1\\ -3 & h\end{pmatrix}\begin{pmatrix}k & t & x\\ -1 & 2 & -3\end{pmatrix}=\begin{pmatrix}-7 & 6 & -5\\ 5 & t & 3k\end{pmatrix}$ [16]

19. Simplify each of the following:

(a) $\begin{pmatrix}3 & 5\\ 8 & 4\end{pmatrix}-\begin{pmatrix}2 & 3\\ 1 & -1\end{pmatrix}+\begin{pmatrix}3 & 5\\ 7 & -9\end{pmatrix}$ (b) $\begin{pmatrix}2 & 2x\\ 3 & 3y\end{pmatrix}-\begin{pmatrix}x & y\\ x & -y\end{pmatrix}+\begin{pmatrix}2x & -y\\ 3y & x\end{pmatrix}$

(c) $\begin{pmatrix}2 & 3 & -1\\ 3 & 2 & 5\end{pmatrix}-2\begin{pmatrix}3 & 1 & -3\\ 4 & 1 & 5\end{pmatrix}$ (d) $\begin{pmatrix}2 & 5\\ 3 & -4\\ 7 & -5\end{pmatrix}+\begin{pmatrix}5 & 2\\ 3 & 0\\ -1 & 4\end{pmatrix}$

(e) $\begin{pmatrix}2 & -4 & 3\\ -5 & 7 & -4\end{pmatrix}+\begin{pmatrix}3 & 2 & 1\\ -1 & -4 & -5\end{pmatrix}$ (f) $\begin{pmatrix}1 & 7\\ 6 & 2\\ 3 & -4\end{pmatrix}-\begin{pmatrix}3 & -2\\ -4 & 1\\ 3 & 5\end{pmatrix}$ [12]

20. Solve the following matrix equations:

(a) $\mathbf{X}+\begin{pmatrix}2 & 1\\ 4 & -3\end{pmatrix}=\begin{pmatrix}3 & 5\\ 7 & -9\end{pmatrix}$ (b) $\mathbf{Y}-\begin{pmatrix}2 & -4\\ -5 & -6\end{pmatrix}=\begin{pmatrix}1 & 7\\ 9 & 3\end{pmatrix}$

(c) $\mathbf{Z}+2\begin{pmatrix}3 & 1\\ -4 & 5\end{pmatrix}=\begin{pmatrix}6 & -10\\ 12 & 3\end{pmatrix}$ (d) $\mathbf{P}-3\begin{pmatrix}1 & -3\\ 4 & -2\end{pmatrix}=2\begin{pmatrix}5 & 7\\ 3 & -4\end{pmatrix}$

(e) $\mathbf{Q}+3\begin{pmatrix}1 & 2 & 5\\ 4 & -6 & 7\end{pmatrix}=\begin{pmatrix}4 & 7 & -9\\ -13 & 12 & 21\end{pmatrix}$ (f) $\mathbf{R}-2\begin{pmatrix}-1 & -3 & -9\\ 5 & 8 & -10\end{pmatrix}=\begin{pmatrix}7 & 3 & -5\\ 8 & -9 & 4\end{pmatrix}$ [12]

21. Evaluate each of the following matrix products where it exists:

(a) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ 2 & -5 \end{pmatrix}$ (c) $\begin{pmatrix} 7 & 9 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 1 & 3 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & -3 \\ 4 & 7 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 7 & 9 \end{pmatrix}$ (e) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ (f) $\begin{pmatrix} 1 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(g) $\begin{pmatrix} 3 \\ 1 \end{pmatrix}\begin{pmatrix} 2 & 4 \end{pmatrix}$ (h) $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (i) $\begin{pmatrix} 3 & 1 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

(j) $\begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}\begin{pmatrix} 1 & 5 & 7 \end{pmatrix}$ (k) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 7 & 8 & 9 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ (l) $\begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$

(m) $\begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ (n) $\begin{pmatrix} 1 & 4 \\ -1 & -3 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 & 4 \\ -3 & -1 & -2 \end{pmatrix}$ [28]

22. Evaluate each of the following matrices where possible.

(a) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}\begin{pmatrix} 4 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 6 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ [4]

23. (a) Given that $\begin{pmatrix} 3 & 0 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$, find the value of $x + y$.

(b) Given that the square of the matrix $\begin{pmatrix} 2 & k \\ 0 & 0 \end{pmatrix}$ is $\begin{pmatrix} 4 & -8 \\ 0 & 0 \end{pmatrix}$, find the value of k.

[4]

24. (a) Evaluate $\begin{pmatrix} -2 & 3 & 1 \end{pmatrix}\begin{pmatrix} -4 \\ 6 \\ 5 \end{pmatrix}$.

(b) Given that $\begin{pmatrix} 2 & -3x \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ 1 \end{pmatrix} = \begin{pmatrix} -8 \\ y \end{pmatrix}$, find the values of x and y. [4]

25. Given that $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 3 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} x & 0 \\ 2 & 1 \end{pmatrix}$. Find the value of x when $\mathbf{AB} = \mathbf{BA}$.

[3]

26. Given that $\begin{pmatrix} 3 & 5 \\ 4 & 8 \end{pmatrix}\begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$, find the value of h and of k. [2]

Answers

1. (a) $\begin{pmatrix} -2 & 8 \\ -3 & 12 \end{pmatrix}$ **(b)** $\begin{pmatrix} 20 \end{pmatrix}$

2. (a) $\frac{1}{2}\begin{pmatrix} -17 & -43 \\ 35 & 21 \end{pmatrix}$ **(b)** $\begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$

3. 2

4. s = -8, t = 2

5. a = 24, b = 12, c = 8, x = 36

6. x = 1, $y = \frac{7}{13}$, $z = -1\frac{7}{13}$

7. 15

8. $a = -4\frac{1}{3}$, b = 3

9. $h = -1\frac{3}{4}$, k = 2, m = 16

10. $\mathbf{A}^2 = \begin{pmatrix} -2 & -2 \\ 6 & 6 \end{pmatrix}$, $\mathbf{A}^3 = \begin{pmatrix} -4 & -4 \\ 12 & 12 \end{pmatrix}$, $\mathbf{A}^5 = \begin{pmatrix} -16 & -16 \\ 48 & 48 \end{pmatrix}$, $\mathbf{A}^7 = \begin{pmatrix} -64 & -64 \\ 192 & 192 \end{pmatrix}$

11. x = 2, y = 3 or x = 3, y = 2

12. (a) a = 0, b = – 4 (b) x = 12, y = 8

(b) a = – 3, b = 2, $c = 3\frac{1}{2}$ (d) a = – 1, b = 12, c = 9, d = 15

(e) a = – 2, b = 0, $c = -1\frac{1}{4}$

13. (a) Yes (b) Yes (c) Yes (d) No

(e) Yes, distributive law (f) No

14. (a) **A** (b) **B** (c) **C** (d) **A**

(e) **C** (f) **B** (g) **B** (h) **C**

16. (a) $a = 3, b = 19$ (b) $a = 2, b = 14\frac{1}{3}$

17. (a) $p = q = 3$ (b) $a = 1, b = 3\frac{1}{2}$ (c) $x = 0, z = 1$

(d) $x = y - 2$ (e) $x = -\frac{1}{2}, y = -3$ (f) $x = 5, y = -2\frac{1}{2}$

18. (a) $a = -4, b = 24, c = 17, d = -2$

(b) $a = -5, b = 6, c = 0, h = -2, k = -6, t = -5$

(c) $x = -1, y = 1\frac{2}{3}, h = -7, k = 3, t = 8$

(d) $a = 0, b = 3\frac{2}{3}$

(e) $p = 3, q = 2, r = 10, s = 7$

(f) $a = 1, b = 5$

(g) $a = -1, b = 5, c = 1, d = -16$

(h) $h = 4, k = -3, t = 2, x = -1$

19. (a) $\begin{pmatrix} 4 & 7 \\ 14 & -4 \end{pmatrix}$ (b) $\begin{pmatrix} 2+x & 2x-2y \\ 3-x+3y & 4y+x \end{pmatrix}$ (c) $\begin{pmatrix} -4 & 1 & 5 \\ -5 & 0 & -5 \end{pmatrix}$

(d) $\begin{pmatrix} 7 & 7 \\ 6 & -4 \\ 6 & -1 \end{pmatrix}$ (e) $\begin{pmatrix} 5 & -2 & 4 \\ -6 & 3 & -9 \end{pmatrix}$ (f) $\begin{pmatrix} -2 & 9 \\ 10 & 1 \\ 0 & -9 \end{pmatrix}$

20. (a) $\begin{pmatrix} 1 & 4 \\ 3 & -6 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 3 \\ 4 & -3 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & -12 \\ 20 & -7 \end{pmatrix}$

(d) $\begin{pmatrix} 13 & 5 \\ 18 & -14 \end{pmatrix}$ (e) $\begin{pmatrix} 1 & 1 & -24 \\ -25 & 30 & 0 \end{pmatrix}$ (f) $\begin{pmatrix} 5 & -3 & -23 \\ 18 & 7 & -16 \end{pmatrix}$

21. (a) $\begin{pmatrix} 13 \\ 19 \end{pmatrix}$ (b) $\begin{pmatrix} 8 & -7 \\ 21 & -20 \end{pmatrix}$ (c) NA

(d) $\begin{pmatrix} -15 & -17 \\ 61 & 83 \end{pmatrix}$ (e) NA (f) (7)

(g) $\begin{pmatrix} 6 & 12 \\ 2 & 4 \end{pmatrix}$ (h) NA (i) (8)

(j) $\begin{pmatrix} 2 & 10 & 14 \\ 5 & 25 & 35 \\ -1 & -5 & -7 \end{pmatrix}$ (k) NA (l) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

(m) $\begin{pmatrix} 3 & 5 \\ 1 & 18 \\ 6 & 10 \end{pmatrix}$ (n) $\begin{pmatrix} -11 & -2 & -4 \\ 8 & 1 & 2 \\ 2 & 6 & 12 \end{pmatrix}$

22. (a) $\begin{pmatrix} 8 & 2 \\ 12 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 3 \end{pmatrix}$

23. (a) 5 (b) -4

24. (a) (31) (b) $x = 8$, $y = -6$

25. $x = -\frac{1}{3}$

26. $h = 4$, $k = -2$

Chapter 6

Secondary 3 Mathematics
Chapter 6 Application of Mathematics in Practical Situations

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 132)

$11\frac{1}{9}\%$ improvement

Just For Fun (pg 151)

59 years old

Secondary 3 Mathematics
Chapter 6 Application of Mathematics in Practical Situations

GENERAL NOTES

Most of the topics in this chapter were formally in the Sec 1 syllabus. One big difference is that the percentage profit or loss must be qualified as whether it is based on the cost price or selling price. This is due to the fact that many companies reported their profit as percentage of their revenue, i.e. based on their sales. Other subtopics that pupils will be introduced here are the simple and compound interests, hire purchase, money exchange, taxation, interpretation of tables and charts.

Money exchange is one topic that is very close to many pupils as more and more Singaporeans travel abroad. You can ask pupils to find out the various exchange rates offered by the various money changers or banks. Why is it that there is a difference between the selling and buying rate. Would it be better if the ASEAN countries adopted a common currency as the European countries have done? Will this work for the ASEAN countries?

It is good to introduce students to some of the financial knowledge at this age. The following are possible projects for the pupils to work on.

(a) Compare the various home loan packages offered by 6 banks for a home loan of $300 000 to be paid over 20 years.

(b) Compare the various car loan packages offered by 6 financial institutions for a car loan of $50 000 to be paid over 7 years. (For brand new car)

(c) Compare the various car loan packages offered by 6 financial institutions for a car loan of $50 000 to be paid over 7 years. (For used car)

(d) If you have $50 000 and wish to put it into a fixed deposit at one of the banks, find out which of the banks offer the best rate.

(e) We can also ask pupils to invest $100 000 on foreign currencies for six months and see who will emerge as winner. We can appoint a few group leaders for the pupils to help calculate transactions that pupils make through the months when they want to switch their investment. The following is a worksheet for reference.

Money Exchange

Each of you are to help invest $100 000 in foreign currencies over the next six months. You are to use your foresight and prediction so as to select the best currency/currencies that will add value to the original sum.

The following are the rates of currencies for your reference.

CURRENCY	RATE AT Oct 13 2006		RATE AT Nov 23 2006		INTEREST RATE
	Buying	Selling	Buying	Selling	
US dollar	1.5740	1.5920	1.5450	1.5630	5.5
Sterling pound	2.9260	2.9580	2.9570	2.9910	4.2
Australian dollar	1.1820	1.1960	1.1970	1.2110	5.8
New Zealand $	1.0340	1.0570	1.0340	1.0570	6.7
Canadian dollar	1.3890	1.4050	1.3530	1.3690	5.5
Malaysian RM	0.4297	0.4304	0.4268	0.4273	3.7
Euro	1.9760	1.9980	1.9990	2.0220	4.2
Chinese Reminbi	0.1990	0.2030	0.1970	0.2010	5.2
Thai baht	0.04190	0.04270	0.04220	0.04300	7.7
Japanese Yen	0.013196	0.013341	0.013232	0.013380	0.2

You are to make careful investments based on the above information. Enter into the following table your investments based on the exchange rates quoted on (23/11/2006) and the interest rates given.

CURRENCY	AMOUNT INVESTED	SING$ EQUIVALENT	INTEREST EARNED	AMOUNT AT 23/11/2006	SING$ EQUIVALENT

You may modify the above table to take in the latest figures and set your own conclusion such as the investment can only be reviewed at the end of month etc.

(f) We can also introduce pupils to the stock market by giving each of them $50 000 virtual money to invest in the Singapore stock market. Form groups of 5 pupils with a leader to decide which stock to invest and record the profit or loss made in every transaction, to include a 0.5% brokerage for every trade. They are to base their trade on the losing prices of the day by referring to the newspaper quoted prices the next day.

(g) The interest rate charged by credit card companies for outstanding amounts due to them is at an annual rate of 24%. Get the students to work out the amount due if $5000 is not paid for 5 years.

(h) As Singaporeans are allowed to invest their CPF money for shares and unit trusts etc, you can ask pupils to find out the percentage of people who make money investing in stock market, unit trust etc as compared with keeping their money in CPF earning 2.5% interest.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: 35 min

Class: _____ Marks: 10

Secondary 3 Multiple-Choice Questions
Chapter 6 Application of Mathematics in Practical Situations

1. Abel bought a mini hi-fi set for $600. He sold it to Bob at a loss of 20%. Bob sold it to Charles and made a profit of 5%. How much did Charles pay for it?
(**A**) $456 (**B**) $504 (**C**) $684 (**D**) $750 (**E**) $756 ( )

2. A man bought x balloons at y cents each. He sold all of them at z cents each. If x, y and z are all increased by 10%, find the percentage increase in profit.
(**A**) 10% (**B**) 15% (**C**) 21% (**D**) 30% (**E**) 40% ( )

3. After the price of fuel went up by 10%, a man reduced his fuel consumption by 10%. What is the percentage change in his fuel bill?
(**A**) decreased by 1% (**B**) increased by 1% (**C**) increased by 9%
(**D**) decreased by 9% (**E**) unchanged ( )

4. The simple interest on $680 for 5 years is $119. What is the rate of interest per annum?
(**A**) 3.5% (**B**) 7% (**C**) 14% (**D**) 42% (**E**) 49% ( )

5. A dealer allows 30% discount on his list prices and then makes a profit of 25% on his cost price. What is the list price of a camera on which he gains $91?
(**A**) $76 (**B**) $109.20 (**C**) $148 (**D**) $520 (**E**) $650 ( )

6. A mixture of coffee is made of grade A and grade B coffee powder in equal parts by weight. Grade A coffee costs $20 per kg and grade B costs $40 per kg. At what price per kg must the mixture be sold to make a profit of 10%?
(**A**) $30 (**B**) $31 (**C**) $33 (**D**) $36 (**E**) $66 ( )

7. The number of pupils in a school increases by 15% each year. If there are 1058 pupils this year, what was the enrolment for the year before last?
(**A**) 920 (**B**) 800 (**C**) 900 (**D**) 1 000 (**E**) 1 028 ( )

8. The length and breadth of a cube are measured 10% too big and the height is measured 10% too small. What is the resulting percentage error in the volume?
(**A**) 30 more (**B**) 30 less (**C**) 10 more
(**D**) 8.9 more (**E**) 8.9 less ( )

9. Each year a car depreciates by $22\frac{1}{2}\%$ of its value at the beginning of the year. What will be the value of a car at the end of two years if its value at the beginning of the first year is \$80 000?

(**A**) \$62 000 (**B**) \$48 050 (**C**) \$40 000
(**D**) \$18 000 (**E**) \$40 500 ( )

10. A Filipino trader exported 7 908 692 pesos worth of goods to Singapore. If the exchange rate was S\$4.8702 to 100 pesos, estimate how much the importer in Singapore paid for the goods in S\$.

(**A**) 40 000 000 (**B**) 350 000 (**C**) 400 000 ( )
(**D**) 4 000 000 (**E**) 320 000

Answers

1. B	2. C	3. A	4. A	5. E
6. C	7. B	8. D	9. B	10. C

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: ☐

Secondary 3 Mathematics Test
Chapter 6 Application of Mathematics in Practical Situations

1. If 15 kg of rambutans cost $25.00, calculate
   (a) the cost of 24 kg of rambutans , [2]
   (b) the quantity of rambutans (in kg) that can be bought for $30.00. [2]

2. A shopkeeper bought an article for $400. By selling it at a discount of 18% of the list price, he made a profit of $22\frac{1}{2}\%$ on the cost. Find the list price. [3]

3. A car depreciates in value by 12% during the first year and by 5% of its value during each succeeding year. If its initial value was $125 000, find
   (a) its value at the end of three years, [3]
   (b) the percentage value of the initial value at the end of three years, giving your answer correct to the nearest whole number. [2]

4.(a) A shopkeeper usually sells VCD players for $360 each. He disposes of a display set by reducing the price by $12\frac{1}{2}\%$. Calculate the selling price of the display set.[2]
   (b) When he sells the VCD player at the usual price of $360, the shopkeeper makes a profit of $16\frac{2}{3}\%$. Calculate the price paid by the shopkeeper. [2]
   (c) Find the percentage profit the shopkeeper makes on the display set. [2]

5. Peter bought some pens for $72. On checking, he found that 8 pens were defective. He sold the remaining pens at 30% above the cost price and made a profit of $16.40. How many dozens of pens did he buy? [3]

6. In a city, electricity is charged at \$0.15 per unit for the first 440 units and \$0.20 for each subsequent unit.
   (a) In June, Mrs Foong used 640 units of electricity.
   What was her electricity bill for the month of June? [2]
   (b) Her electricity consumption for the month of July was reduced by $22\frac{1}{2}\%$. What was the corresponding percentage decrease in her bill? [3]

7. (a) A manufacturer produced 18 000 pens and sold them in packets of 5 at \$1.60 per packet. Calculate the total selling price. [2]
   (b) The cost of production consisted of expenditure on administration, labour and materials. The cost of administration was \$2 100. Labour and materials cost 15 cents per pen. Calculate the total cost of production and express the profit made as a percentage of this total cost. [4]
   (c) A further 45 000 of these pens were produced and sold at the same price of \$1.60 per packet of 5 pens. In this case, the profit made was $33\frac{1}{3}\%$ of the cost of producing them. Calculate the cost of producing the 45 000 articles. [3]

8. (a) The perimeter of a quadrilateral is 132 cm and the sides are in the ratio 5:8:15:17. Calculate how much longer is the longest side when compared to the shortest side. [2]

   (b) John heard the sound of a gun 3.8 seconds after it was fired. Taking the speed of sound to be 330 metres per second, calculate the distance between John and the gun, giving your answer in kilometres. [2]

9. Mrs Tay, Mrs Chen and Mdm Rosnah each decided to buy a photocopying machine that was priced at \$ 6 400.
   (a) Mrs Tay offered her old machine in part exchange and the salesman allowed her $27\frac{1}{2}\%$ off the cost of the new machine. Calculate how much more Mrs Tay had to pay for her new photocopying machine. [2]
   (b) Mrs Chen paid for the new photocopying machine in cash and was given a discount. Given that she paid \$5 488 for her new photocopying machine, calculate the percentage discount she received. [2]
   (c) Mdm Rosnah agreed to pay 45% of \$6 400 as a deposit and the balance in equal monthly instalments over a period of two years. Given that each monthly installment is \$176, calculate how much Mdm Rosnah paid for her photocopying machine altogether. [2]
   (d) The salesman had hoped to sell each new photocopying machine for \$6 400 so that he would make a profit of 28% on the cost price. Calculate the cost price of each new machine. [2]

10. The amounts of money Peter, Paul and Jane have respectively are in the ratio of 3:5:7. If Jane gives Paul $198, the ratio of the amounts of money Paul and Jane have becomes 7:5. How much money do Peter, Paul and Jane have altogether? [3]

11. A bicycle shop bought 20 bicycles for $4 000. On checking, the shopkeeper found that 5 of the bicycles were slightly damaged. He sold each of the remaining bicycles at the normal selling price and each of the slightly damaged bicycles at $\frac{3}{4}$ of the normal selling price. What was the normal selling price of each bicycle if he made 35% profit on the whole transaction? [4]

12. In a trapezium, the ratio of the lengths of the two parallel sides is 3:2. While keeping the height of the trapezium constant, the length of the shorter parallel side is increased by 30% and the length of the longer side is decreased by 30%. What is the percentage increase or decrease in the area of the trapezium? [4]

13. The selling price of a television set is $854. Mr Koh paid cash for a set and was given 10% discount. Mr Chow bought a set from the same shop by installments and was charged 12% more than the selling price. Find the difference between the prices Mr Koh and Mr Chow paid for their television sets. [3]

14. A credit card company charges 2.5% interest per month on amounts not paid in previous months. If Mr Lee pays off $145.00 of his Novmber account of $336.82, how much interest charges will be added to his December account? [2]

15. John has a passbook account which earns him 2.5% per annum for minimum monthly balances up to $5 000 and 3.25% per annum for balances over $5 000. Calculate the December interest John receives if the minimum monthly balances in the 6 months up to December were: $5 428, $4 906, $4 269, $5 548, $4 946 and $5 967. [5]

16. A 29-inch colour television set is advertised at $1 999 at full price or $67.50 per month,
    (a) If $67.50 per month is the only amount to be paid, how many months approximately will it take to pay off the television set? [2]
    (b) If the actual terms stated are a 10% deposit and 36 monthly payments of $67.50, how much is the total cost of the television set? [3]
    (c) How much more than the full price should be paid to buy the television set on terms? [1]

17. Payroll tax is due from companies paying wages. It is calculated as follows:
4% of payroll for amounts up to \$500 000 and $5\frac{1}{2}\%$ of payroll for amounts greater than \$500 000. Calculate the payroll tax due on the following payrolls:
(a) \$357 000
(b)\$950 000 [2]

18. Mortgage duty on money borrowed to purchase a property is calculated as follows: \$75 on the first 150 000, plus 0.6% of the remaining amount. Calculate the mortgage duty payable on the following loans:
(a) \$125 000 (b) \$560 000 [3]

19. The rate of commission on the sale of a house is 4% on the first 150 000, 2% on the next 450 000, $1\frac{1}{2}\%$ on the next 400 000 and 1% thereafter. A real estate agent sells two properties during the month of June with values of \$587 500 and \$1 209 500.
(a) Calculate the commission due on each property. [4]
(b) What was the total commission received from the sale of the two properties? [1]
(c) If the agent receives 25% of the total commission received from the sales, how much does the agent earn in June? [2]

20. A man invests a sum of money at simple interest. Each year he receives \$128 interest on the amount invested. When the rate of interest rises by 0.75%, the annual interest is raised to \$140. Calculate the sum of money invested and the new percentage rate of interest. [5]

21. Coffee powder costing \$9 per kg is mixed with coffee powder costing \$13.50 per kg. The mixture is sold at \$12 per kg, thereby making 20% profit.
Find the ratio in which the two types of coffee powder are mixed. [4]

22. A shopkeeper sold an article for \$160 making a 20% loss. How much must the shopkeeper sell the article to make a profit of 20%? [2]

23. Sugar costing \$1 per kg is mixed with sugar costing \$1.20 per kg in the ratio 3:1. Find the selling price per kg of the mixture to make a 20% gain. [3]

24. Peter has \$30 000 for investment. He can choose to deposit the sum of money in the bank which pays a simple interest of $7\frac{1}{2}\%$ per annum or invest in a building society which pays simple interest at a rate of \$7.50 for every \$1 000 invested per month. Find the difference in interests paid by the bank and the building society after 1 year. [3]

25. Mr Kwan borrowed from a finance company to buy a second-hand car. He pays back the loan through monthly installments for a period of 4 years. In the first year, each monthly instalment is \$1 000. For each subsequent year, the monthly installment is reduced by 10% from the previous year. Find the amount of money Mr Kwan paid for the car. [4]

26. Three persons, A, B and C, enter into a business together by contributing \$50 000, \$45 000 and \$60 000 for periods of 10 months, 12 months and 8 months respectively. At the end of the year, the gross profit is \$100 000 and the expenses amount to 24% of the gross profit. The net profit is shared among A, B and C in proportion to their contributions. Find the profit A receives. [5]

27. A shopkeeper bought a watch for US\$60 and sold it for S\$102.60. Find his percentage profit.
(Take US\$1=S\$1.425). [3]

28. Mr Beaver travels from A to B on a bus at a speed of 40 km/h and then walks from B to C at a speed of 5 km/h. Given that the distance between A and C is 55 km and he takes 4 hours for the whole journey, find the ratio of the distance between B and C. [4]

29. It takes 3 men and 5 women to complete a job in 17 days and it takes 5 men and 3 women to complete the same job in 15 days. What is the ratio of the work rates of a man and a woman? [5]

30. It takes 40 workers to complete half of a certain job in 15 days. If the remaining job needs to be completed in 12 days, how many workers need to be deployed? [2]

31. Raymond drives from Town A to Town B, 120 km apart, at an average speed of 50 km/h. Find his average speed on the return journey if the round trip takes 5 hours and 24 minutes altogether. [2]

32. Mrs Chua makes stuffed toys and is paid according to the following differential rates:

0-80 toys	\$3.10 per toy
81-150 toys	\$3.75 per toy
151-200 toys	\$4.60 per toy
201(or over)toys	\$5.80 per toy

During a month when Mrs Chua makes 242 toys, how much does she earn? [3]

33. Simon places \$3 684 in an investment account earning 8.25% per annum from May 28th to August 27th (including those days). Calculate the interest earned. [3]

34. (a) A train leaves Town A at 09 00 hours and is scheduled to reach Town B at 12 30 hours. What is its average speed if the distance is 273 kilometres? [2]
(b) On a certain day, the train travels at this speed for 2 hours and is then delayed for three-quarters of an hour. If its average speed is 60 km/h for the rest of the journey, when does it arrive at Town B? [3]

35. The daily wages of a skilled worker, Kevin, and an apprentice, John, were \$45 and \$18 respectively. Working separately, Kevin received \$900 when he had completed a piece of work and John received \$540 when he had finished the same type of work. If they did the work together, and assuming that they were working at their own individual rates, how much would each receive when the work was completed? [5]

36. An alloy of zinc and tin contains 33% of zinc by weight. Find the weight of zinc which must be added to 600 kg of this alloy if the final percentage of tin is to be $33\frac{1}{3}$. [4]

37. In what proportion must a chemist mix two solutions of a certain chemical which cost him \$24 and \$36 per litre respectively so that by selling the mixture at \$35 per litre, he is able to make 25% profit on his outlay? [3]

38. Goods in a shop are marked at 35 % above the cost price. What profit per cent is made if 20 % is taken off for cash? What is the greatest percentage that can be taken off without causing a loss to the shopkeeper? [5]

39. During a sale, a shopkeeper allows customers a discount of 0.175 in the dollar on the marked price which originally gave him a 40% profit. What does the customer pay now for goods which cost the shopkeeper \$1 400? [3]

40. (a) Find a man's taxable income if he paid \$1 567.50 in tax when the income tax was levied at $12\frac{1}{2}\%$.

(b) If S\$1 = US\$0.56 and S\$1 = M\$1.57, find how many Malaysian ringgits can be exchanged for US\$600. Give your answer correct to the nearest dollar. [4]

41. (a) A man buys a book for \$12. His advertised selling price is 25% higher, but he gives a discount of 12% to schools. What is the selling price of the book to schools?

(b) A salesman receives a basic salary of \$550 and commission of 4% of the value of goods sold in excess of \$4 500. Find his income in a month when he sells goods worth \$18 000. [4]

42. Mr. Chen borrows a sum of money for 5 years. For the first 3 years, the simple interest, at a rate of 8% per annum, amounts to \$3 600. For the next 2 years, the simple interest is $10\frac{1}{2}\%$ per annum. Calculate

(a) the amount of money he has borrowed,

(b) the amount he has to pay back altogether at the end of 5 years. [4]

43. (a) A man spends 10% of his monthly income on rent, 15% on food, 12% on clothes, 8% on income tax, 21% on other expenses and saves the rest. Given that he saves \$1 292 a month, find his monthly income.

(b) The value of a car depreciates each year by 15% of its value at the beginning of the year. If a brand new car costs \$56 000, find its value at the end of 4 years correct to the nearest 100 dollars. [4]

44. A travelling salesman receives a basic salary of \$800 a month and a commission equal to 4% of the value of goods sold plus a car allowance of 60¢ per km.

(a) Find his income for a particular month when he sells goods worth \$13 500 and travels 800 km.

(b) The next month, he travels 996 km and receives a total income of \$1 970. Calculate the percentage increase in the value of goods sold. [4]

45. (a) Mr Chen is entitled to a tax-free allowance of \$16 000 and he pays tax at 12% on his income over that figure. Calculate how much he has to pay if his income is \$38 000.

(b) Mr Lin is entitled to a tax-free allowance of \$13 000 and he pays tax at 14%. Find his income if he pays \$4 060 in tax. [4]

46. A manufacturer knows that 6% of the light bulbs he makes are defective. Find the number of bulbs he must produce in order to obtain 611 light bulbs which are not defective.

The manufacturing cost for the light bulbs is \$586.56. If he sells the non-defective light bulbs at a profit of 25%, find the selling price of each light bulb. [4]

47. (a) The interest on a man's investment increases from $8\frac{3}{4}\%$ to $11\frac{1}{4}\%$ per annum. Find the value of his investment if his annual income from it increases by \$75.

(b) The cost of manufacturing a car is \$7 800. Find the selling price of the car if it is sold at a profit of $17\frac{1}{2}\%$ on the cost price. [5]

48. A man buys a flat for \$100 000 and rents it out. He puts 14% of each month's rent aside for repairs and maintenance of the flat; pays \$272 in taxes per year and realises $8\frac{1}{2}\%$ on his investments. Calculate the monthly rent, correct to the nearest dollar. [3]

Answers

1. (a) $40
   (b) 18kg

2. $578.20

3. (a) 99 275
   (b) 79%

4. (a) $315
   (b) $300
   (c) 5%

5. 12

6. (a) 106
   (b) 27.2%

7. (a) $5 760
   (b) $4 800, 16.7%
   (c) $10 800

8. (a) 35.2 cm
   (b) 1.254 km

9. (a) $4 640
   (b) 14¼ %
   (c) $7 104
   (d) $5 000

10. $1 485

11. $288

12. 6% decrease

13. $187.88

14. $4.80

15. $75.31

16. (a) 30
(b) $2 629.90
(c) $630.90

17. (a) $14 280
(b) $52 250

18. (a) $75
(b) $2 535

19. (a) $14 # 750, $23 095
(b) $37 845
(c) $9 461.25

20. $1 600, 8.75%

21. 7:2

22. $240

23. $1.26

24. $450

25. $41 268

26. $25 000

27. 20%

28. 8 : 11

29. 5 : 3

30. 50

31. 40 km/h

32. $984.10

33. $76.61

34. (a) 78 km/h
(b) 13 42

35. $540, $216

36. 606 kg

37. 2 : 1

38. 8%, 25.9%

39. $1 617

40. (a) $12 540 (b) 1682 Malaysian ringgits

41. (a) $13.20 (b) $1 090

42. (a) $15 000 (b) $21 750

43. (a) $3 800 (b) $29 200

44. (a) $1 820 (b) 6%

45. (a) $2 640 (b) $42 000

46. 650 light bulbs, $1.20

47. (a) $3 000 (b) $9 165

48. $850

Chapter 7

Secondary 3 Mathematics
Chapter 7 Linear Graphs and Their Applications

ANSWERS FOR ENRICHMENT ACTIVITIES

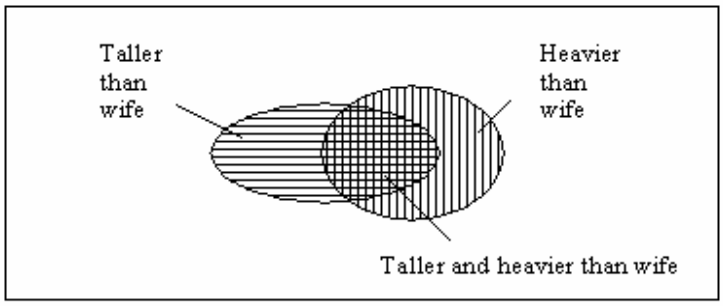
Just For Fun (pg 173)

Suppose 32 rabbit heads were counted . The number of legs would have been $32 \times 4 = 128$.
This is $128 - 100 = 28$ legs more than those counted.
We can assume that these extra legs belong to the chickens and therefore, there are $28 \div 2 = 14$ fewer rabbits.
$\therefore$ the number of chickens counted = 14 and the number of rabbits counted $= 32 - 14 = 18$.
Therefore, there are 14 chickens and 18 rabbits.

Just For Fun (pg 176)

Let x be the number of husbands who are taller than their wives, y be the number of husbands who are heavier than their wives and z be the number of husbands who are taller and heavier than their wives. (A *Venn diagram* clarifies the problem.)

Then $z = \frac{2}{3}x, z = \frac{3}{4}y$ and 1 000- x - y + z = 120 giving x = 720.



Just For Fun (pg 183)

Let the distance for the whole journey be d km and Chonglin's speed be x km/h.

Then we have $\frac{d}{x} - \frac{d}{x+1} = \frac{3}{60}$ and $\frac{d}{x} - \frac{d}{x+6} = \frac{15}{60}$. Hence, the solution is d = 30.

Secondary 3 Mathematics
Chapter 7 Linear Graphs and Their Applications

GENERAL NOTES

As an introduction to the topic, the teacher may spend a little bit of time introducing and discussing the many situations that make use of the idea of co-ordinate geometry. Some of the examples not mentioned in the text are:

(1) The latitude and longitude of a place on earth, i.e. map work.
(2) The seating layout in cinemas, stadiums, etc. with which most students should be familiar.
(3) The display of flashcards during the National Day celebrations. Each card bearer is given a row and a column number for the co-ordinator to supervise.
(4) Before a mural is painted on a wall, a picture is normally drawn on a piece of grid paper and then transferred to the wall.

Bring their attention also to the relationship between the graphs of $y = x + c$ and $y = x$ and in general to the relationship between the graphs of $y = mx + c$ and $y = mx$, i.e. the graph of $y = x + c$ is the translation of the graph of $y = x$, c units up or down parallel to the x–axis depending on whether $c > 0$ or $c < 0$ and the graph of $y = mx + c$ is the translation of the graph of $y = mx$, c units up or down parallel to the x–axis depending on whether $c > 0$ or $c < 0$. Thus, lead students to the conclusion that the graphs of $y = mx + c$ for various values of c are parallel and cut the y–axis at different points corresponding to different values of c.

Emphasise the fact that when two quantities are related in any way, it is often useful to show the relationship by means of a graph and the purpose of a graph is to convey information visually and quickly. Stress also that a good graph must be neat, clear and concise.

Conversion graphs are something new to the students for at primary level, they were exposed only to travel graphs. A conversion graph is also known as a "ready reckoner". A "ready reckoner" relating the cost of a copper pipe to its length may be found on a chart of a building material supplier. Using this graph, a salesman can quickly determine the cost of any length of copper piping. Besides the type of conversion graphs discussed in the textbook, teachers may like to get students to suggest other types of everyday situations where conversion graphs may be useful. The ***Dynamic Mathematics Series*** on "**The Business of GRAPHS**" will provide extra drill and practice for the pupils if your school do have these CDs.

Choice of scale is important in this chapter. A scale will be determined by the *biggest* and *lowest* values of a given variable. The scale should be as large as possible as this allows space for more details. Plotting or graphing should be done carefully and neatly. The scale and names of the quantities (along the respective axes) should be clearly specified. Students must also be reminded of the following precautions when reading off figures from any graph:

- Check the scale
- Check the starting point of the respective axes
- Use a ruler

XYZ SECONDARY SCHOOL

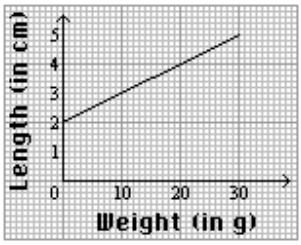
Name: _____ ( ) Date: _____

Time allowed: min

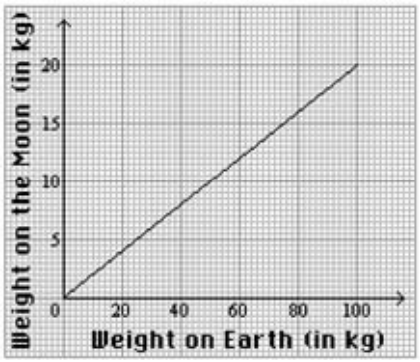
Class: _____ Marks: /

Secondary 3 Mathematics Test
Chapter 7 Linear Graphs and Their Applications

1. The diagram shows the different lengths of a spring when different weights are attached to it.
   (a) What is the original length of the spring? [1]
   (b) What is the length of the spring when the attached weight is 12 g? [1]
   (c) Find the attached weight if the spring is extended by 2.4 cm from its original length. [1]



2. The graph shows the different weights of an object on Earth and on the Moon.
   Use the graph to find the weight of an object
   (a) on Earth if it weighs 14 kg on the Moon, [1]
   (b) on the Moon if its weight on earth is 48 kg. [1]



3. The graph shows the cost in dollars of printing various numbers of name cards. From the graph, find

(a) the cost of printing

(i) 100 cards [1]

(ii) 150 cards [1]

(iii) 300 cards [1]

(b) the number of cards that could be printed for

(i) $100 [1]

(ii) $250 [1]

(iii) $300 [1]

4. The table below displays the distance travelled by a car and the amount of petrol remaining in the tank.

Distance travelled (in km)	0	20	40	80	160	320
Amount of petrol left (in litres)	50	48	46	42	34	18

(a) Draw a graph to show the relationship between the distance covered and the amount of petrol in the tank, using suitable scales. [2]

(b) When the car has travelled 50 km, how much petrol is left in the tank? [1]

(c) The car starts with a full tank of 50 litres. How far has the car travelled before the tank is empty? [1]

(d) If the car travels at an uniform speed throughout, what is the rate of consumption of petrol? [2]

(e) How far has the car travelled when only 21 litres of petrol remain in the tank? [1]

5. It is given that 1 kg of soya beans is required to produce 6 litres of soya bean milk. If x denotes the weight of soya beans needed and y the amount of soya bean milk produced, draw a graph to illustrate the relationship between x and y. [2]

Use your graph to find

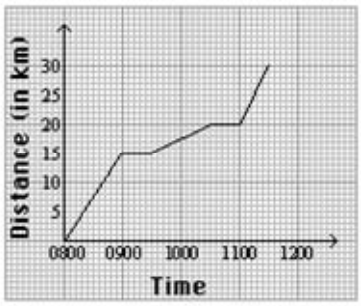
(a) the amount of soya bean milk produced by

(i) 4 kg of soya beans (ii) 6.5 kg of soya beans [2]

(b) the weight of soya beans needed to produce

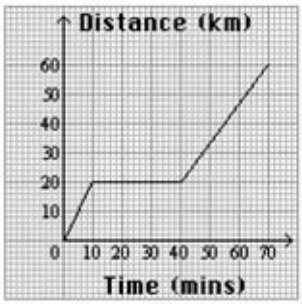
(i) 12 litres of soya bean milk (ii) 21 litres of soya bean milk [2]

6. The distance-time graph shows the graph of Joshua's journey when he goes to visit his teacher.
   (a) How far away does his teacher live? [1]
   (b) What is the fastest speed at which he travelled? [1]
   (c) How long did he rest altogether? [1]

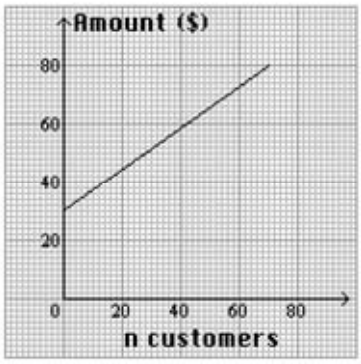


7. It is given that US$10 was equivalent to S$15 in July 1997. Draw a graph to show the relationship between American and Singapore dollars up to US$100. Use your graph to find the conversion of the following:
   (a) *American* dollars into *Singapore* dollars
   (i) US$16 (ii) US$72 [2]
   (b) *Singapore* dollars into *American* dollars
   (i) S$30 (ii) S$96 [2]

8. The diagram shows the distance-time graph of a motorist after leaving home. Using the graph, find
   (a) the speed in km/h, during the last 30 minutes. [2]
   (b) the average speed, in km/h, for the whole journey. [2]



9. A restaurant owner pays a waiter an amount of $*A* per day. The amount is made up of a fixed daily wage plus a variable amount, which depends on the number of customers he serves. The graph illustrates the relationship between $*A* and *n*, which is the number of customers he serves.
Use the graph to find
(a) the fixed daily wage [1]
(b) the amount of money he received in a day when he served 50 customers [1]
(c) the number of customers he served on the day he received $46 [1]



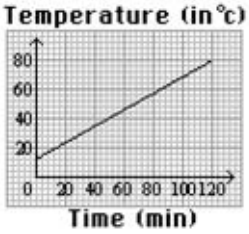
10. The maximum number of marks in an examination is 60. The table below shows the relationship between the actual marks obtained by some candidates and their percentage marks.

Actual Marks	0	30	60
Percentage Marks	0	50	100

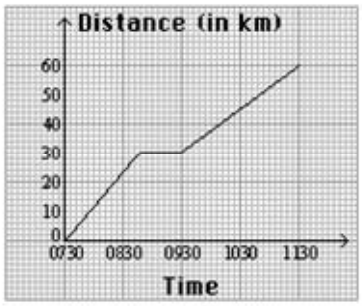
Using suitable scales, draw a graph to convert the actual marks to percentage marks.
From your graph, find
(a) the percentage marks obtained by a candidate with
(i) 15 marks (ii) 48 marks [1]
(b) the actual marks of a candidate whose percentage marks are
(i) 35 (ii) 55 [1]

11. The graph shows the change in the temperature of the water in a hot water tank after the heater is switched on.
Use your graph to find
(a) the temperature of the water in the tank after
(i) 28 minutes (ii) 1 hour 24 minutes [2]

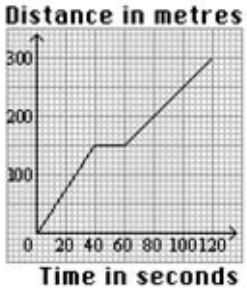
(b) the time taken for the temperature to reach
(i) 32° C (ii) 76° C [2]



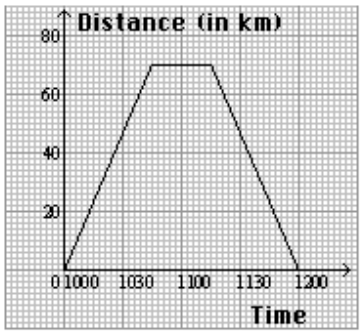
12. The diagram shows the distance-time graph of Mr Chew.
(a) How far did he travel? [1]
(b) What is his speed for the first hour? [1]
(c) Find his average speed for the whole journey. [2]



13. The diagram shows the travel graph of a moving body after leaving a starting point. Using the graph, find
(a) the time interval during which the body is stationary, [1]
(b) the speed during the last minute, [2]
(c) the average speed for the whole journey in km/h. [3]



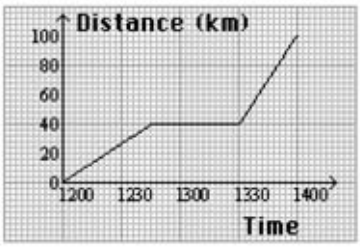
14. The diagram shows the travel graph of a car. Using the graph, find
    (a) the speed of the car for the first part of the journey, [1]
    (b) the duration during which the car stopped, [1]
    (c) the average speed of the car for the whole journey, [2]
    (d) the fastest speed of the car during the journey. [1]



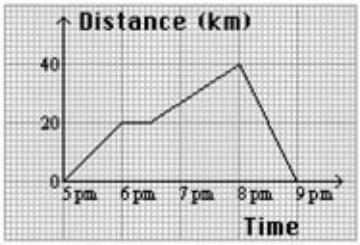
15. Given that 36 km/h is equal to 10 m/s, construct a graph for converting km/h to m/s. [3]
    Use your graph to convert (a) 55 km/h to m/s, [1]
    (b) 108 m/s to km/h. [1]

16. Given that 5 kg =11 pounds, construct a graph for converting pounds to kg and vice versa. [3]
    Use your graph to convert align (a) 66 pounds to kg, [1]
    (b) 45 kg to pounds. [1]

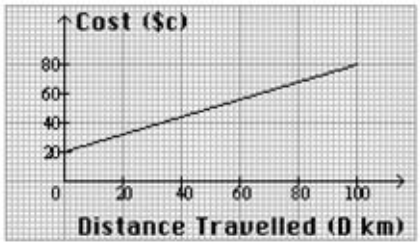
17. The diagram shows the distance-time graph of a motorist.
    (a) How far did he travel in the first hour? [1]
    (b) What was his average speed during the first hour? [1]
    (c) What is his speed during the last leg of his journey? [1]
    (d) What is his average speed for the whole journey? [1]



18. The graph shows the journey of Peter who leaves home at 5 p.m.
    (a) What is Peter's average speed for the outward journey? [1]
    (b) How many hours was Peter away from home? [1]
    (c) How far did Peter travel in total? [1]
    (d) What is Peter's average speed for the entire journey? [1]



19. The graph shows the cost ($*C*) of hiring a van to travel *D* km from a rental company. Use the graph to find
    (a) the cost of hiring a van to travel
    (i) 34 km, [1]
    (ii) 68 km. [1]
    (b) the distance travelled if the cost of hiring a van is
    (i) $52 [1]
    (ii) $32 [1]



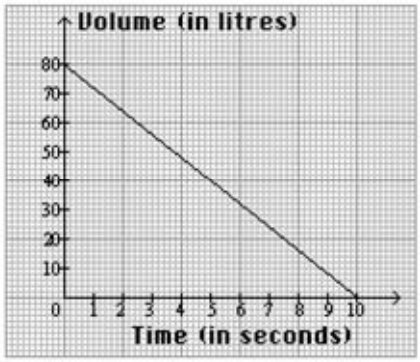
20. Given that 10 gallons = 45 litres, construct a graph to convert gallons to litres and vice versa.
    Use your graph to
    (a) convert into litres
    (i) 4 gallons (ii) 6 gallons
    (b) convert into gallons
    (i) 21.6 litres (ii) 39.6 gallons [4]

21. It is given that S$1 was equivalent to 20 Thai baht in July 1997. Draw a graph to show the relationship between the Singapore dollar and the Thai baht up to 200 Thai baht. [2]
    Use your graph to find the conversion of the following:
    (a) Singapore dollars into Thai baht
    (i) $5.50 (ii) $8.20 [2]
    (b) Thai baht into Singapore dollars
    (i) 140 baht (ii) 88 baht [2]

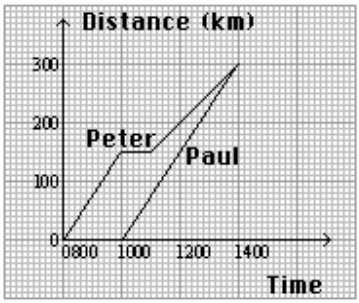
22. It is given that S$55 was equivalent to M$100 in July 1997. Draw a graph to show the relationship between the Singapore dollar and the Malaysian ringgit up to M$100. Use your graph to convert
(a) into Malaysian ringgit
(i) S$22 (ii) #S$45 [2]
(b) into Singapore dollars
(i) M$60 (ii) M$82 [2]

23. It is given that HK$10 was equivalent to ¥160 in July 1997. Draw a graph to show the relationship between Hong Kong dollars and Japanese yen up to ¥800. Use your graph to convert into
(a) Japanese yen
(i) HK$25 (ii) HK$42
(c) Hong Kong dollars
(i) ¥480 (ii) ¥640 [4]

24. The graph shows the rate at which the water is emptied from a container.
Use the graph to find
(a) the amount of water in the container at first, [1]
(b) the time taken to empty the container, [1]
(c) the volume of water in the container after 3 seconds, [1]
(d) the time taken to empty $\frac{3}{4}$ of the tank. [1]

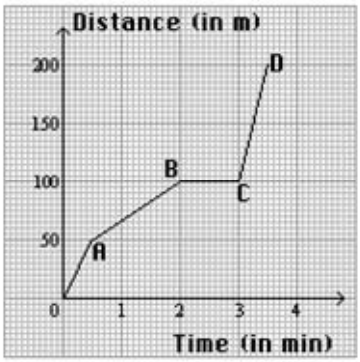


25. The diagram shows the travel graphs of Peter and Paul.
From the graphs, find
(a) Peter's speed in the first two hours, [1]
(b) the distance between them at 12 noon, [1]
(c) Paul's speed throughout the journey, [1]
(d) the difference between their average speeds for the entire journey. [2]

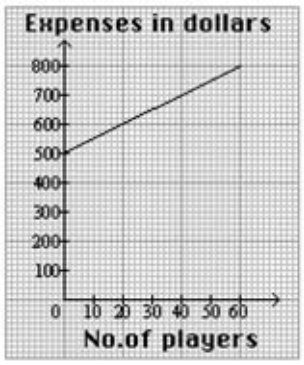


26. It is given that A$75 was equivalent to 100 German marks (DM) in July 1997. Draw a graph to show the relationship between the Australian dollars and German marks up to DM100. Use your graph to convert
(a) into German marks
(i) A$39 (ii)A$63
(b) into Australian dollars
(i) DM60 (ii) DM80 [4]

27. The diagram shows the travel graph of a moving body.
(a) What section would indicate that the body was resting? [1]
(b) Which section shows the body moving at the fastest speed? [2]
(c) Find the average speed in km/h of the body for the entire journey. [1]



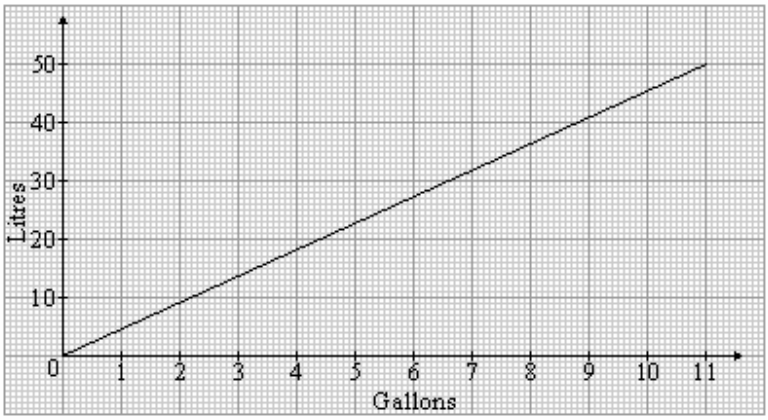
28. The diagram shows the relationship between the expenses of a basketball tournament and the number of players attending the tournament. From your graph, find
(a) the expenses when the number of players is
(i) 12 (ii) 48 [2]
(b) the difference in the number of players when the expenses differ by 100 dollars. [1]



29. It is given that S$24 was equivalent to 100 French francs in July 1997. Draw a graph to show the relationship between the Singapore dollars and the French franc for up to 100 francs.
Use your graph to find the conversion of the following:
(a) Singapore dollars into French francs
(i) S$8 (ii) S$18
(b) French francs into Singapore dollars
(i) 25 francs (ii) 80 francs [4]

30. Given that 10 m/s is equivalent to 36 km/h, draw a conversion graph using the following scales:
Horizontal: km/h, 0 to 150, 1 cm to 10 km/h
Vertical: m/s, 0 to 40, 2 cm to 10 m/s
Use your graph to change to
(a) m/s
(i) 72 km/h (ii) 126 km/h
(b) km/h
(i) 15 m/s (ii) 40 m/s [4]

31.



The diagram above shows the graph for converting gallons to litres.

(a) A motorist bought 5.2 gallons of petrol. Given that 1 litre of petrol costs $1.50, use the conversion graph to calculate how much the motorist paid for petrol.

(b) Given that 1 gallon of milk costs $11.25, use the graph to calculate the cost of 18 litres of milk. [4]

32. Given that 20 m/s is equivalent to 72 km/h, draw a conversion graph using the following scales:

Horizontal: km/h, 0 to 100, 1 cm to 10 km/h
Vertical: m/s, 0 to 30, 2 cm to 10 m/s

Use your graph to change to

(a) m/s
(i) 36 km/h (ii) 55 km/h (iii) 75 km/h

(b) km/h
(i) 5 m/s (ii) 25m/s (iii) 14m/s [8]

33. Draw a conversion graph to convert marks out of 65 to percentage marks using suitable scales. Use your graph to convert the following marks out of 65 to percentage marks:
(a) 6 (b) 26 (c) 39 (d) 54 (e) 63 [8]

34. On a certain day in 2006, the exchange rate for US$80 was S$130. Draw a graph to convert American dollars to Singapore dollars.
Use your graph to
(a) change to Singapore dollars
(i) US$30 (ii) US$48 (iii) US$69 (iv) US$150
(b) change to American dollars
(i) S$21 (ii) S$85 (iii) S$112 (iv) S$270 [10]

35. In an experiment, a rat was placed at different distance d (in cm) from a goal box and the pull p (in g) of the rat towards the food placed in the goal box was measured.

The formula connecting p and d is $p = -\frac{1}{5}d + 70,\ 30 \le d \le 175$.

(a) Draw the graph of the equation $p = -\frac{1}{5}d + 70,\ 30 \le d \le 175$.

(b) Use your graph to find the value of p when the value of d is
(i) 35 (ii) 66 (iii) 125 (iv) 160 [7]

36. The formula connecting the cost, \$ C, in producing n television sets is $C = 96\,000 + 80n$.

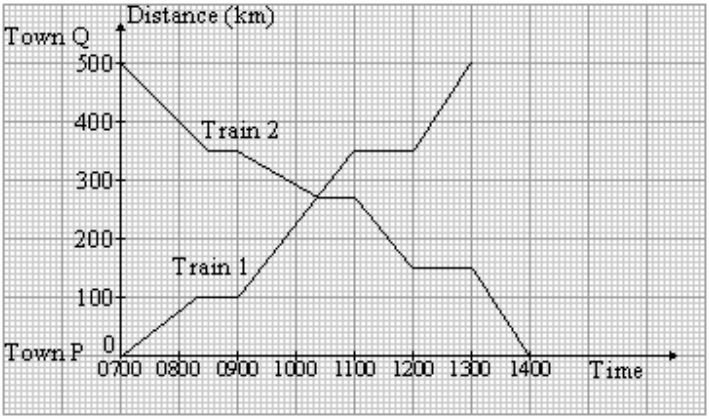
(a) Draw the graph of this equation for $0 \le n \le 1000$.

(b) Use your graph to find the cost for producing
(i) 50 (ii) 125 (iii) 650 (iv) 800
television sets.

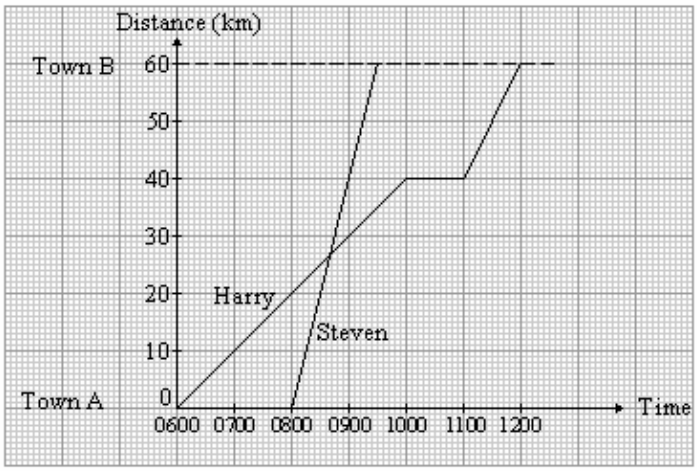
(c) Given that m television sets cost \$132 000 to produce, use your graph to find the value of m. [7]

37. Train 1 travels from Town P to Town Q, a distance of 500 km apart. It leaves Town P at 07 00 and arrives at Town Q at 13 00. Train 2 leaves Town Q at 07 00, arriving at Town P at 14 00. The diagram below shows the distance-time graphs of the two trains. Find

(a) the distance travelled in the first two hours by each train,
(b) the speed of each train before the first stop,
(c) when and where the two trains passed each other,
(d) the average speed of each train for the entire journey. [6]



38.



Harry leaves Town A at 06 00 to travel to Town B, 60 km away. Two hours later, Steven leaves Town A to travel to Town B too. The graph above shows the distance they had travelled in a given time. Find

(a) how long Steven took to reach Town B and his speed for the journey,
(b) when and where Steven overtook Harry,
(c) how far Harry had travelled after 1 hour,
(d) how far Harry had travelled when Steven reached his destination,
(e) how long Steven took to travel 45 km,
(f) Harry's average speed for the whole journey. [8]

Answers

1. (a) 2cm

   (b) 3.2 cm

   (c) 24g

2. (a) 70kg
   (b) 9.5kg

3. (a) (i) $150 (ii) $200 (iii) $350
   (b) (i) 50 cards (ii) 200 cards (iii) 250 cards

4. (b) 45 litres (c) 500 km (d) 0.1 litre per km (e) 290 km

5. (a) (i) 24 litres (ii) 39 litres
   (b) (i) 2 kg (ii) 3.5 kg

6. (a) 30 km (b) 20 km/h (c) 1 hour

7. (a) (i) S$24 (ii) S$108
   (b) (i) US$20 (ii) US$64

8. (a) 80 km/h
   (b) 51 3/7 km/h

9. (a) $30 (b) $66 (c) 22 customers

10. (a) (i) 25 (ii) 80
    (b) (i) 21 (ii) 33

11. (a) (i) 28 °C (ii) 60 °C
    (b) (i) 36 minutes (ii) 112 minutes

12. (a) 60 km (b) 24 km/h (c) 15 km/h

13. (a) 20 seconds (b) 2.5 m/s (c) 9 km/h

14. (a) 93 1/3 km/h (b) ½ hour (c) 70 km/h (d) 120 km/h

15. (a) 198 m/s (b) 30 km/h

16. (a) 30 kg (b) 99 pounds

17. (a) 40 km (b) 40 km/h (c) 120 km/h (d) 50 km/h

18. (a) 13 1/3 (b) 4 hours (c) 80 km (d) 20 km/h

19. (a) (i) $40 (ii) $60
(b) (i) 54 km (ii) 20 km

20. (a) (i) 18 litres (b) 27 litres
(b) (i) 4.8 gallons (b) 8.8 gallons

21. (a) (i) 110 baht (b) 164 baht
(b) (i) S$7 (b) S$4.40

22. (a) (i) M$40 (ii) M$82
(b) (i) S$33 (ii) S$45

23. (a) (I) ¥ 400 (ii) ¥ 672
(b) (i) HK$ 30 (ii) HK$ 40

24. (a) 80 litres (b) 10 seconds (c) 56 litres (d) 7.6 seconds

25. (a) 75 km/h (b) 50 km (c) 75 km/h (d) 25 km/h

26. (a) (i) DM 52 (ii) DM 84 (b)(i) A$45 (ii) A$60

27. (a) *BC* (b) *CD* (d) 3 3/7 km/h

28. (a) (i) $560 (b) $740
(b) 20

29. (a) (i) 33 francs (ii) 75 francs
(b) (i) S$6 (ii) S$ 19

30. (a) (i) 20 m/s (ii) 35 m/s
(b) (i) 54 km/h (ii) 144 km/h

31. (a) $36 (b) $45

32. (a) (i) 10 (ii) 15 (iii) 21

(b) (i) 18 (ii) 90 (iii) 50

33. (a) 9 (b) 40 (c) 60 (d) 83 (e) 97

34. (a) (i) 49 (ii) 78 (iii) 112 (iv) 244

(b) (i) 13 (ii) 52 (iii) 69 (iv) 166

35. (b) (i) 63 (ii) 57 (iii) 45 (iv) 38

36. (b) (i) 100 000 (ii) 106 000 (iii) 148 000 (iv) 160000

(c) 450

37. (a) 100 km, 150 km (b) 80 km/h, 100 km/h

(c) 10 18, 270 km from Town P (d) $83\frac{1}{3}\text{km/h}, 71\frac{3}{7}\text{km/h}$

38. (a) 1 hour 30 min, 40 km/h (b) 08 42, 27 km from Town A (c) 10 km

(d) 35 km (e) 1 hour 6 min (f) 10 km/h

Chapter 8

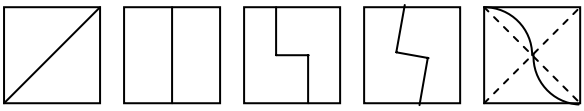
Secondary 3 Mathematics
Chapter 8 Congruent & Similar Triangles

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 206)

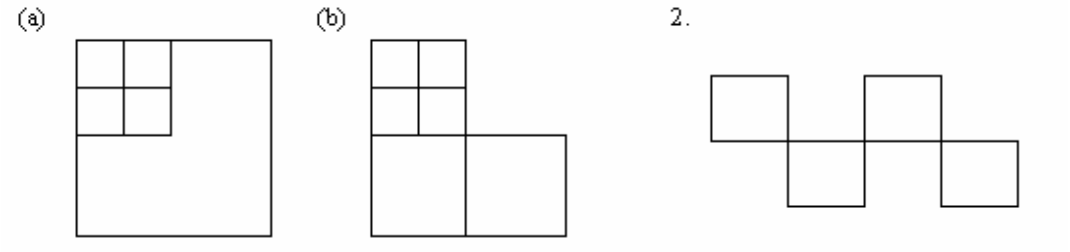
The 6 toothpicks can be joined to form a tetrahedron, thus giving 4 congruent triangles. This is an example to let students view a problem from other non-conventional methods, switching from 2-dimensional to 3-dimensional solutions.

Just For Fun (pg 216)



Just For Fun (pg 219)

1.



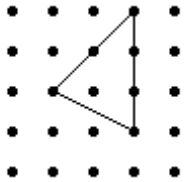
Secondary 3 Mathematics
Chapter 8 Congruent & Similar Triangles

GENERAL NOTES

In this chapter some simple applications of congruent triangles and similar triangles are discussed. (Teachers may ask their students to come up with some other applications.) Many students enjoy field activities in mathematics. They can also try using the methods mentioned in Questions 10 and 11 (pg 233) to determine the height of a tree on school grounds.

The puzzles and exploration at the beginning of the chapter will help students to grasp the concept of congruency. It may be useful here to repeat the way the concept of congruency is used in everyday situations such as in the replacement of worn-out parts with the same part number etc.

In discussing the tests for similarity between two triangles, lead up to the idea that two triangles are similar if they can be made congruent by enlargement or reduction. The geoboard (shown below) can serve as an excellent aid in illustrating this idea. It can also be used to illustrate congruency tests.



For example, teachers can ask their students to form another triangle having sides equal to the three sides of the triangle shown on the right. This can be repeated by having them form a triangle so that the triangle formed and the given triangle have two sides and included angle equal and so on. In each case, students should note whether the triangle formed and the given triangle are congruent.

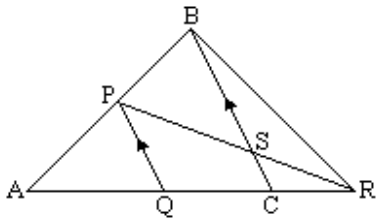
XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: 35 min

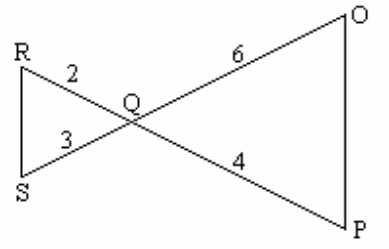
Class: _____ Marks: 9

Secondary 3 Multiple-Choice Questions
Chapter 8 Congruent & Similar Triangles

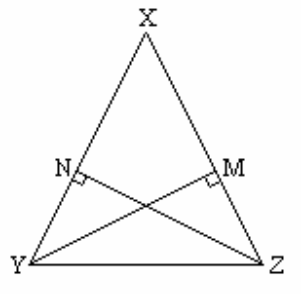


1. In the diagram, PQ is parallel to BC, AP = PB and QC = CR. Given that BC = 8 cm, the length of SC in cm is

   (A) 4 (B) 2 (C) $2\frac{2}{3}$ (D) 6 (E) $5\frac{1}{3}$ ( )

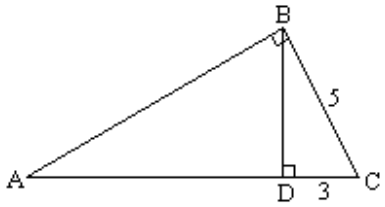


2. In the diagram, PQ = 4 cm, QR= 2 cm, OQ = 6 cm and QS = 3 cm. We say that
   (A) triangles RSQ and POQ are similar.
   (B) triangles SOP and PQO are similar.
   (C) RP and SO intersect at right angles.
   (D) triangles SQP and RQO are congruent.
   (E) O, P, R and S are points on the circumference of a circle. ( )



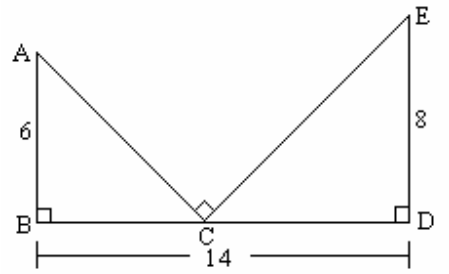
3. In the diagram, $\angle XMY = \angle XNZ = 90°$.
   If XY = XZ, then
   (A) ΔXYZ is equilateral.
   (B) Triangles XMY and XNZ are congruent.
   (C) Triangles XNZ and YMZ are similar.
   (D) $\angle XYZ = 90°$.
   (E) YZ = YM = ZN.

( )



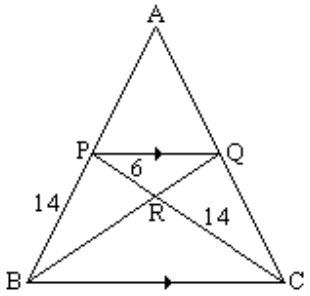
4. In the diagram, $\angle ABC = \angle BDC = 90°$, BC = 5 cm and DC = 3 cm. The length of AD in cm is

   (A) $8\frac{1}{3}$ (B) 5 (C) $5\frac{1}{3}$ (D) 2 (E) 12 ( )



5. In the diagram, $A\hat{B}C = A\hat{C}E = C\hat{D}E = 90°$, AB = 6 cm, BD = 14 cm and DE = 8 cm. Calculate BC.

   (A) 6cm only (B) 7cm only
   (C) 8cm only (D) 9cm only
   (E) 6cm or 8cm ( )

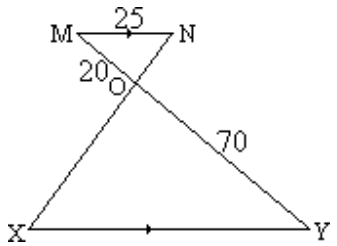


6. In the diagram, PQ // BC. If PR = 6 cm and RC = PB = 14 cm, then AP is

   (A) 6cm (B) $32\frac{2}{3}$ cm (C) 14cm
   (D) $10\frac{1}{2}$ cm (E) 20cm ( )

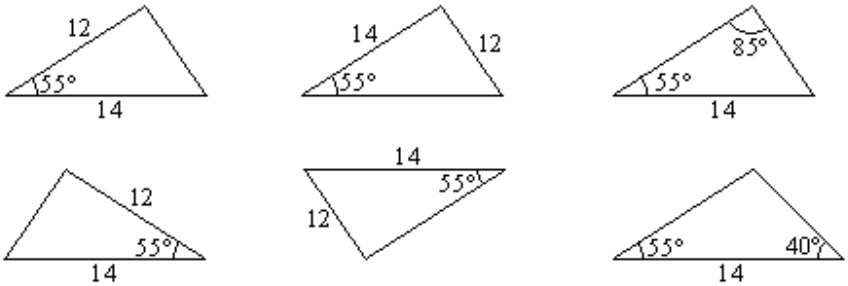
7. In the diagram MN//XY, MN = 25 cm, MO = 20 cm and YO = 70 cm. What is the length of XY?

   (A) 87 cm (B) $87\frac{1}{2}$ cm (C) 88 cm
   (D) $88\frac{1}{2}$ cm (E) $89\frac{1}{2}$ cm

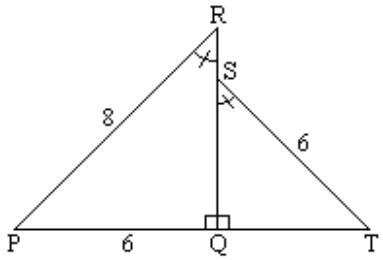


( )

I. II. III.



8. Which of the following cases represent a pair of congruent triangles?
   (A) I only (B) II only (C) III only
   (D) I and II only (E) I and III only ( )



9. In the diagram $P\hat{R}Q = T\hat{S}Q$, $P\hat{Q}R = T\hat{Q}S = 90°$, PR = 8 cm and PQ = ST = 6 cm. Find the length of QT.
   (A) 3cm (B) $4\frac{1}{2}$ cm (C) 6 cm (D) 8 cm (E) $\frac{2}{9}$ cm
   ( )

Answers

1. B	2, A	3. B	4. C	5. A
6. D	7. B	8. E	9. B	

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: ☐

Secondary 3 Mathematics Test
Chapter 8 Congruent & Similar Triangles

1. Copy and complete the following:
In ΔPQR and ΔTSR,

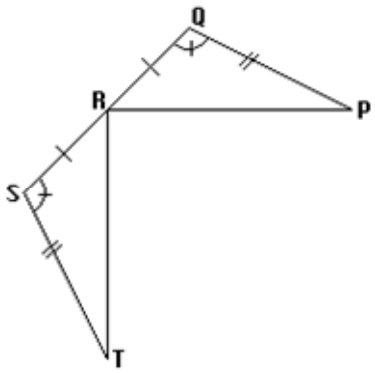
PQ = _____

_____ = $T\hat{S}R$

QR = _____

$\therefore \Delta PQR \equiv \Delta TSR$ (_____)

$\therefore PR$ = _____
and $Q\hat{P}R$ = _____ .

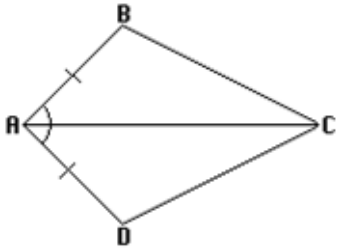


[3]

2. Copy and complete the following:
In ΔABC and ΔADC ,

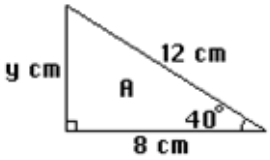
AB = _____
$B\hat{A}C$ = _____
AC = _____
$\Delta ABC = \Delta ADC$ (_____)

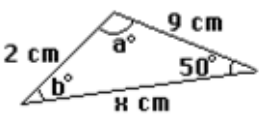
BC = _____
and $A\hat{C}B$ = _____



[3]

3. Given that $\Delta A \equiv \Delta B$, copy and complete the following:
a = _____
b = _____
x = _____
y = _____
z = _____

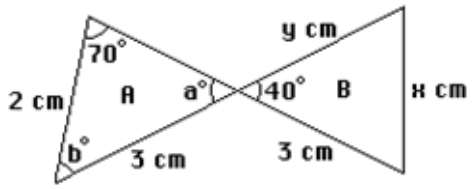




[3]

4. Given that $\Delta A \equiv \Delta B$, copy and complete the following:

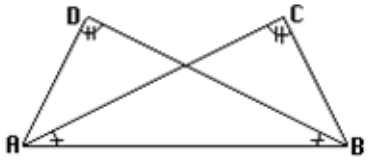
a = _____
b = _____
x = _____
y = _____



[2]

5. Copy and complete the following:
In ΔABC and ΔBAD,

$A\hat{C}B$ = _____
$B\hat{A}C$ = _____
AB = _____
$\therefore \Delta ABC \equiv \Delta BAD$ (_____)
$\therefore AC$ = _____
and BC = _____ .

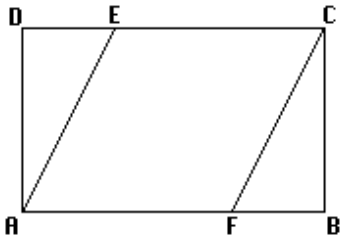


[3]

6. In the diagram, $ABCD$ is a rectangle and AE // FC. Copy and complete the following:
In ΔADE and ΔCBF,

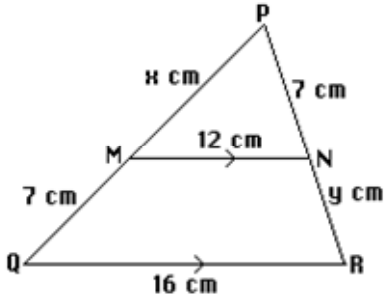
$A\hat{D}E$ = _____
$D\hat{A}E$ = _____
AD = _____

$\Delta ADE \equiv \Delta CBF$ (_____)
DE = _____ and AE = _____ .

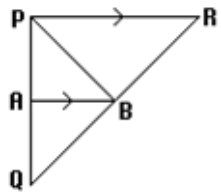


[3]

7. In the figure, ΔPQR and ΔPMN are similar. Find the values of x and y. [4]



8. In the diagram, AB is parallel to PR. Given that QB = 4 cm, BR = 6 cm, PR = 15 cm and the area of $\Delta PBR = 9a$ cm^2, find

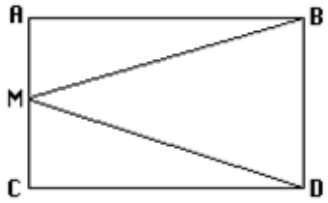


(a) the length of AB

[2]

(b) the area of ΔPQR in terms of a. [2]

9. In the diagram, M is the mid-point of AC, $AMB = DMC$ and $MB = DM$. Name a pair of congruent triangles and state the case for congruency. [2]



10. In the diagram, AD // MC, MD // BC and M is the mid–point of AB. Prove that $\Delta AMD \equiv \Delta MBC$ by copy and complete the following:

$D\hat{A}M$ = _____

$A\hat{M}D$ = _____

AM = _____

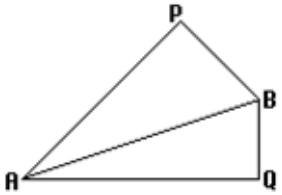
$\Delta AMD = \Delta MBC$ (_____) [4]

11. In the diagram, $AP \perp BP$, $BQ \perp AQ$ and $AP = AQ$. Prove that $\Delta ABP \equiv \Delta ABQ$ by copy and complete the following:

$A\hat{P}B$ = _____ = _____

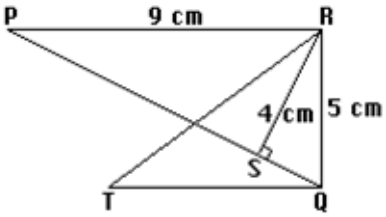
AP = _____

AB = _____

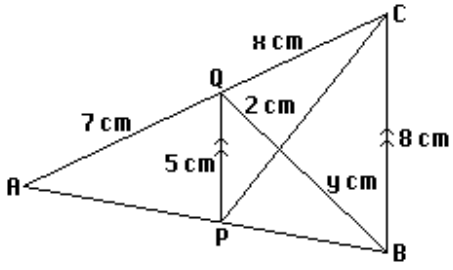


$\Delta ABP \equiv \Delta ABQ$ (_____) [4]

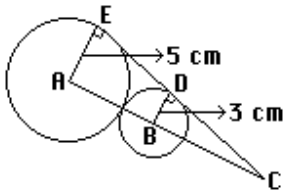
12. In the diagram, given that ΔPRS and ΔQRT are similar, find the length of TR. [2]



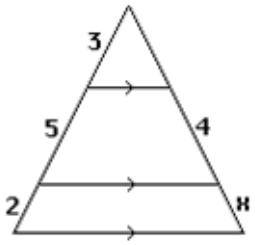
13. In the diagram shown, *PQ* // *BC*. Calculate the values of *x* and *y*. [4]



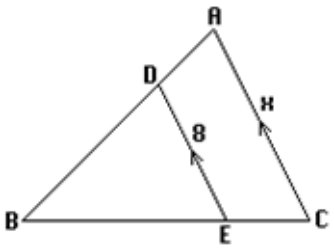
14. In the diagram, the radii of the two circles touching each other are 5 cm and 3 cm respectively. Calculate the length of *BC*. [3]



15. In the diagram, all lengths are given in cm. Find the length of the side marked with the letter x. [4]

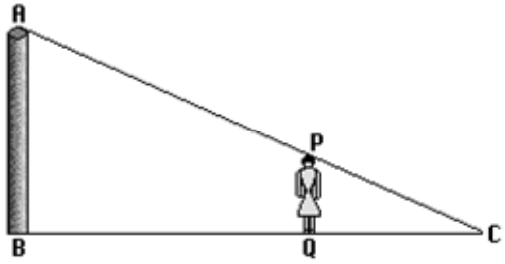


16. In the diagram, *DE* // *AC*. Find
    (a) the lengths of the sides marked with *x* and *y*. [3]
    (b) the ratio of the parameters of ΔABC and ΔDBE. [2]

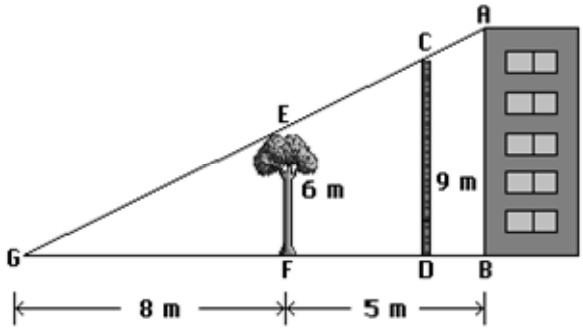


17. At 10 00, a boy and a tree cast shadows of lengths 2.5 m and 15 m respectively. If the boy's height is 150 cm, find the height of the tree. [3]

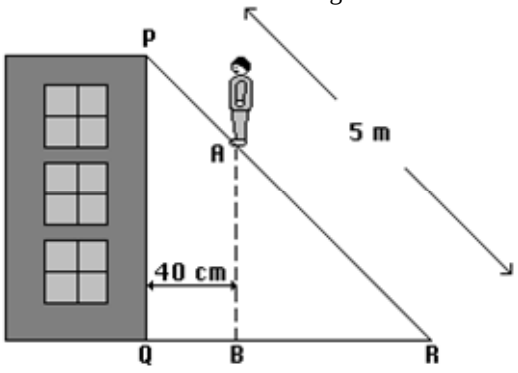
18. The diagram below shows a girl (*PQ*), 120 cm tall, standing at a distance from a 4.8m-tall pole (*AB*).If the shadow of the girl on the ground is 1.5 m, find the distance *BQ* of the girl from the pole. [3]



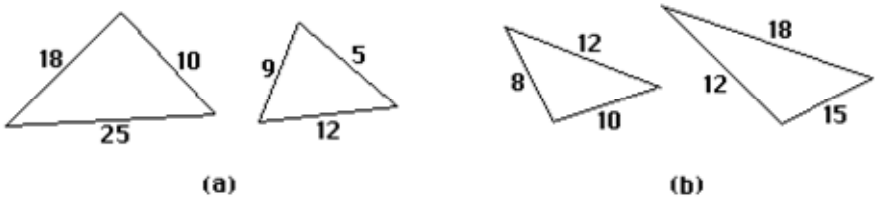
19. The diagram shows a building *AB*, an electrical pole *CD* and a tree *EF* in a row. The tree, 5 m away from the building, is 6 m tall and casts a shadow of 8 m.
Find (a) the height of the building *AB*, [3]
(b) the distance *FD* of the pole from the tree if the pole is 9 m tall. [3]



20. The diagram shows a ladder, 5 m long, reaching the top of a building from level ground. A worker has moved up $\frac{4}{5}$ of the ladder such that his horizontal distance from the building is 40 cm. Find the distance *QR* of the foot of the ladder from the foot of the building. [3]

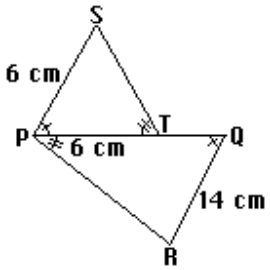


21.

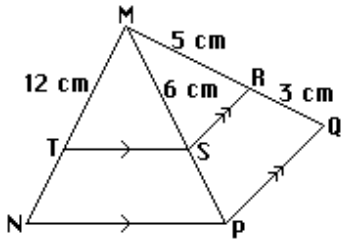


Give a reason why the pair of triangles in (a) are not similar and in (b) are similar. [2]

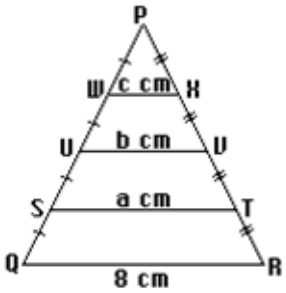
22. In the diagram $S\hat{P}T = P\hat{Q}R$, $S\hat{T}P = Q\hat{P}R$. If PT = 6 cm, QR = 14 cm and PS = 6 cm, find the length of QT. [2]



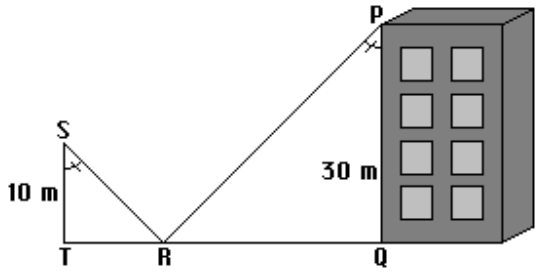
23. In the diagram, TS // NP and SR // PQ. If MR = 5 cm, RQ = 3 cm, MS = 6 cm and MN = 12 cm, find the length of MT. [3]



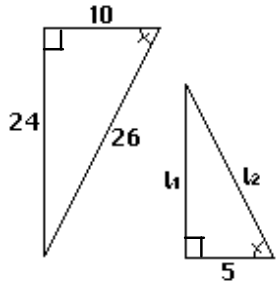
24. In the diagram, $PW = WU = US = SQ$ and $PX = XV = VT = TR$. If QR = 8 cm, $ST = a$ cm, $UV = b$ cm and $WX = c$ cm, find the value of $a + b + c$. [4]



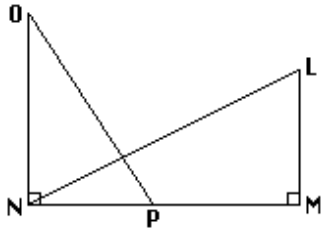
25. The diagram shows a flagpole *ST*, 10 m tall and a building *PQ*, 30 m tall. Alvin stands between the pole and the building at the point *R* such that $R\hat{S}T = R\hat{P}Q$. Find his distance from the flagpole. [3]



26. The diagram shows two similar triangles. Calculate the value of $l_1 + l_2$. [3]

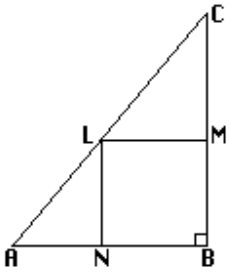


27. In the diagram, *ON* = *NM*, *PN* = *LM* and $O\hat{N}P = L\hat{M}N = 90$.
(a) Prove that $\Delta OPN = \Delta NLM$. [3]
(b) If $L\hat{M}N = 32°$, find $O\hat{P}N$. [1]

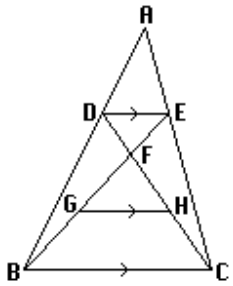


28. In the diagram, *DF* ⊥ *BC*, *DG* ⊥ *AB*, *DE* ⊥ *AE*, *BC* ⊥ *AE*, *DG* = *DE* and *BF* = *CF*.
(a) Name two pairs of congruent triangles and state the property for congruency in each case. [2]
(b) If *HC* = 4 cm, *DE* = 7.2 cm and *CE* = 4 cm, calculate the length of *AE*. [3]

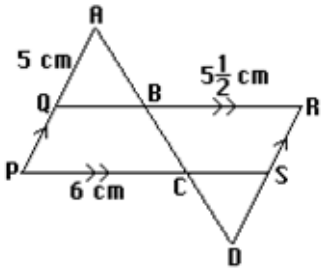
29. In the diagram, *ABC* is a right-angled triangle and *LNBM* is a square. If *BC* = 12 cm and *AB* = $7\frac{1}{5}$ cm, find the area of the square *LNBM*. [4]



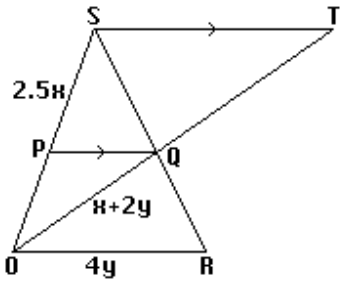
30. In the diagram, *DE* // *GH* // *BC* and *DE* : *GH* : *BC* = 5:7:10.
(a) If *FH* = 21 cm, find the length of *DF*. [1]
(b) If *GB* = 12 cm, find the length of *FG*. [2]



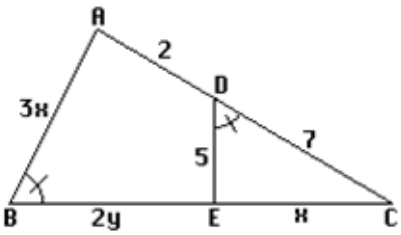
31. In the diagram, *PQRS* is a parallelogram and *ABCD* is a straight line. Given that *AQ* = 3 cm, *QP* = 7 cm, *PC* = 6 cm and *BR* = $5\frac{1}{2}$ cm, calculate the length of (a) *QB* [2]
(b) *CS* [1]
(c) *SD* [3]



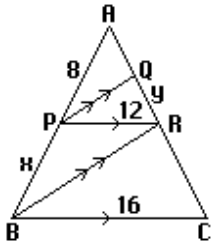
32. In the diagram, OR // PQ // ST.
Given that $OP = x$ units, $PS = 2.5x$ units,
$OR = 4y$ units and $OQ = x + 2y$ units,
express (a) PQ in terms of y, [2]
(b) ST in terms of y, [2]
(c) express QT in terms of x and y. [2]



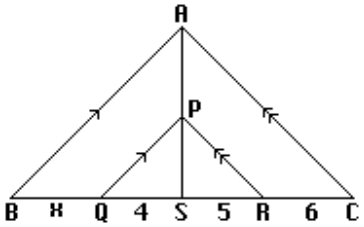
33. In the diagram, $A\hat{B}C = C\hat{D}E$. Find the value of the ratio $x : y$. [3]



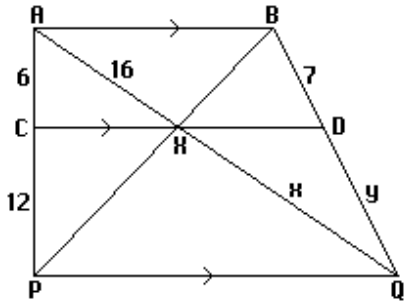
34. In the diagram, PR // BC, PQ // BR, $AP = 8$ units, $PB = x$ units, $PR = 12$ units, $BC = 16$ units, $AR = 6$ units and $QR = y$ units. Calculate the value of the product xy. [6]



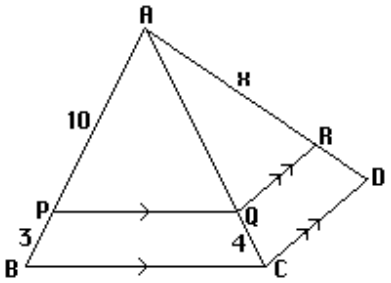
35. In the diagram, AB // PQ, AC // PR, $BQ = x$ units, $QS = 4$ units, $SR = 5$ units and $RC = 6$ units.
Find the value of x. [4]



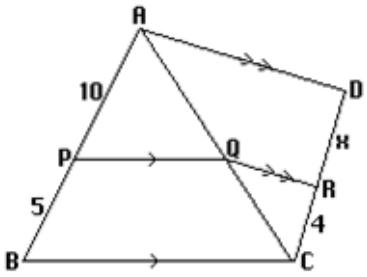
36. In the diagram, AB // CD // PQ, AC = 6 units, CP = 12 units, AX = 16 units, XQ = x units, BD = 7 units and DQ = y units.
Find the value of $x + y$. [6]



37. In the diagram PQ // BC, QR // AD, AP = 10 cm, PB = 3 cm, QC = 4 cm, AR = x cm and AD = 12 cm. Calculate the value of x. [5]



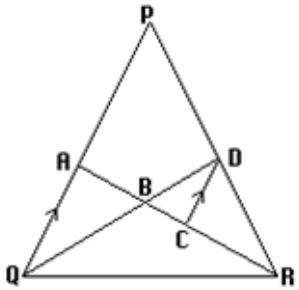
38. In the diagram, PQ // BC, QR // AD, AP = 10 cm, PB = 5 cm, DR = x cm and RC = 4 cm.
Find the value of x. [5]



39. In ΔABC, $PD = \frac{4}{5} PR$ and $PA = \frac{5}{6} PQ$. AR and DQ meets at B and AQ // DC.

Prove that (a) $DC = \frac{1}{6} PQ$, [2]

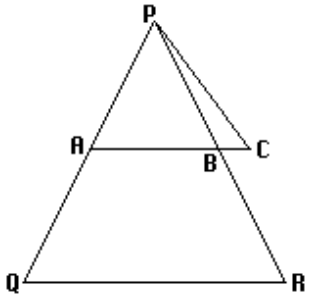
(b) ΔABQ and ΔDBC are congruent. [3]



40. In the diagram, AC // QR. $PQ = PR = 28$ cm, $AC = PC = 20$ cm and $QR = 14$ cm.

(a) Prove that ΔPQR and ΔACP are similar. [2]

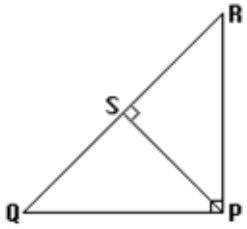
(b) Calculate the length of AQ. [3]



41. In the diagram, PQR is a triangle, right–angled at P. PS is the perpendicular from P to QR.

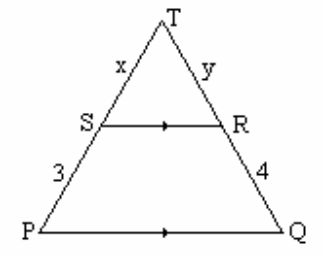
(a) Prove that ΔRSP and ΔPSQ are similar. [3]

(b) Given that $QS = 36$ cm and $RS = 25$ cm, find the length of PS. [2]



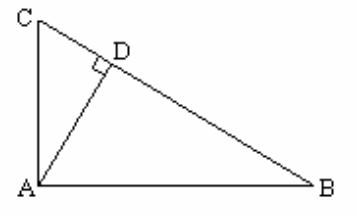
42. The sides of a triangle are 9 cm, 7 cm and 6 cm long. The longest side of a similar triangle is 6 cm. Find the lengths of the other sides of the second triangle. [3]

43. In the diagram, SR is parallel to PQ. SP = 3 cm, TS = x cm, TR = y cm and RQ = 4cm. Write down an equation connecting x and y. Given that PQ = 2SR, write down the values of x and y. [5]

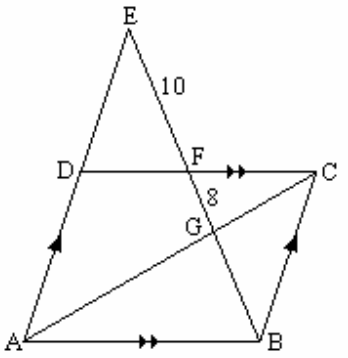


44. A cone has a height of 12 m and a base radius of 5 m. Find the diameter of the circular section cut out from the cone by a plane parallel to the base and 3 m away from it. [3]

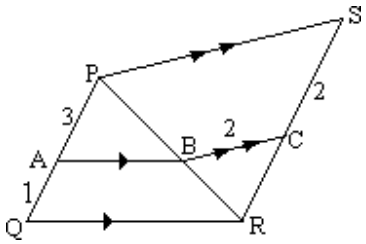
45. ΔABC is right-angled at A. Given that AD is perpendicular to BC, prove that $AB \times AD = BD \times AC$. [4]



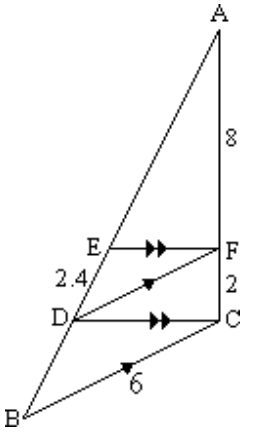
46. In the diagram, ABCD is a parallelogram and EDA and EFGB are straight lines. If EF = 10 cm and FG = 8 cm, calculate the length of GB. [4]



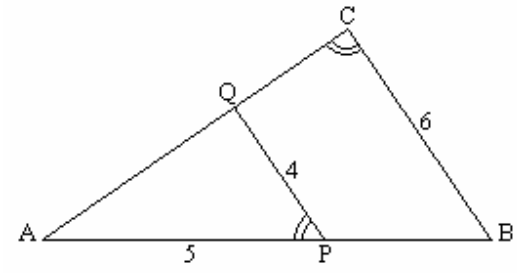
47. In the diagram, AB // QR, BC // PS, PA = 3 cm, AQ = 1 cm and BC = CS = 2cm. Calculate the length of
(a) PS,
(b) RC. [4]



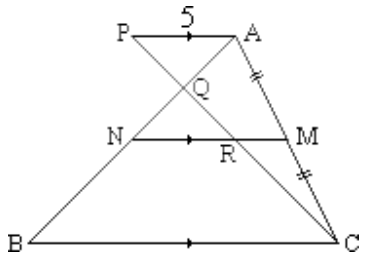
48. In the diagram, DF is parallel to BC, EF is parallel to DC, BC = 6 cm, DE = 2.4 cm, CF = 2 cm and AF = 8 cm. Calculate the length of
(a) AE,
(b) DB,
(c) DF. [6]



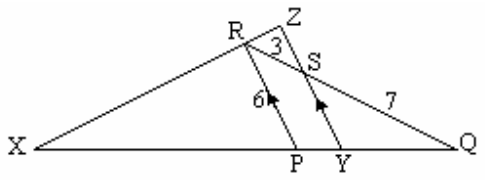
49. In the diagram, $A\hat{C}B = A\hat{P}Q$, AP = 5cm, PQ = 4 cm and BC = 6 cm. Calculate the length of AC. [3]



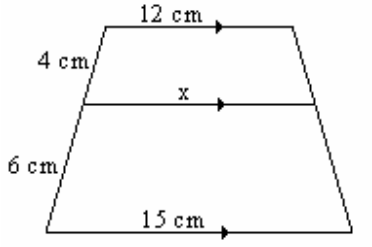
50. In the diagram , PA // NM // BC, M is the mid-point of AC, PA = 5 cm and PC = 4PQ. Find the length of MN. [4]



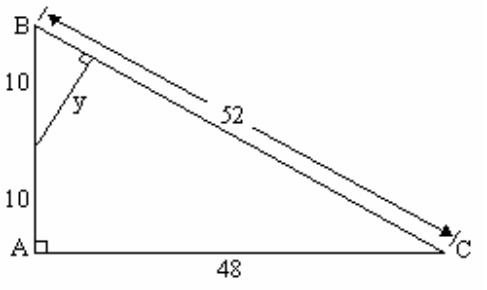
51. In the diagram, PR // YZ, PR = 6 cm, RS = 3 cm and SQ = 7 cm.
(a) Calculate the length of SY.
(b) If XZ = 5RZ, find the length of YZ. [5]



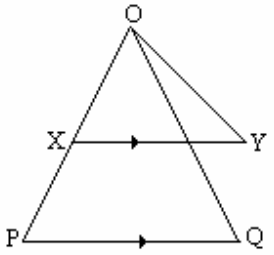
52. Find the value of *x* in the diagram. [4]



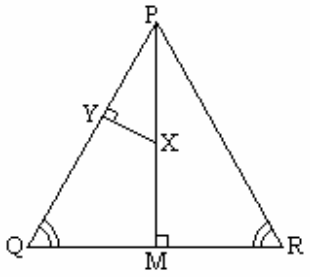
53. Find the value of *y* in the diagram. [4]



54. In the diagram, XY is parallel to PQ. OP = OQ = 24 cm, XY = OY = 16 cm and PQ = 9 cm.
    (a) Prove that triangles OPQ and YXO are similar.
    (b) Calculate the length of XP. [5]



55. In the diagram $P\hat{Q}R = P\hat{R}Q$, PM is perpendicular to QR and XY is perpendicular to PQ. Given that PQ = 13 cm, QR = 10 cm, PM = 12 cm and XY = $2\frac{1}{2}$ cm, find the length of QY. [4]



Answers

1. $TS,\ P\hat{Q}R,\ SR,\ SAS,\ TR,\ S\hat{T}R$

2. $AD,\ D\hat{A}C,\ AC,\ SAS,\ DC,\ A\hat{C}D$

3. $a = 90,\ b = 40,\ x = 12,\ y = 9,\ z = 8$

4. $a = 40,\ b = 70,\ x = 2,\ y = 3$

5. $B\hat{D}A,\ A\hat{B}D,\ AB,\ AAS,\ BD,\ AD$

6. $CBF,\ BCF,\ BC,\ AAS,\ BF,\ CF$

7. $x = 21,\ y = 2\frac{1}{3}$

8. (a) 6 cm (b) $15a\ \text{cm}^2$

9. $\Delta AMB \equiv \Delta CMD,\ SAS$

10. $C\hat{M}B,\ M\hat{B}C,\ MB,\ AAS$

11. $A\hat{Q}B,\ 90°,\ AQ,\ AB,\ RHS$

12. $11\frac{1}{4}$ cm

13. $x = 4.2,\ y = 3.2$

14. 4 cm

15. 1.6 cm

16. (a) $x = 11.2,\ y = 4$ (b) 7 : 5

17. 9 m

18. 4.5 m

19. (a) $9\frac{3}{4}$ m (b) 4 m

20. 2 m

21. $\frac{18}{9} = \frac{10}{5} = 2 \neq \frac{25}{12}$ $\frac{8}{12} = \frac{12}{18} = \frac{10}{15} = \frac{2}{3}$

22. 8 cm

23. 7.5 cm

24. 12

25. 5 m

26. 25

27. (b) 58°

28. (a) $\Delta BFD \equiv \Delta CFD$, *SAS*; $\Delta AGD \equiv \Delta AED$, *RHS*
(b) 9 cm

29. $20\frac{1}{4}$ cm^2

30. (a) 15 cm (b) 28 cm

31. (a) $2\frac{1}{2}$ cm (b) 2 cm (c) 4 cm

32. (a) $\frac{20}{7}y$ units (b) $10y$ units (c) $\frac{5}{2}x + 5y$ units

33. 5: 8

34. 4

35. 4.8

36. 46

37. 9.2

38. 8

40. (b) 18 cm

41. (b) 30 cm

42. $4\frac{2}{3}$ cm, 4 cm

43. $4x = 3y$; $x = 3$, $y = 4$

44. $7\frac{1}{2}$ m

46. 12 cm

47. (a) 8 cm (b) $\frac{2}{3}$ cm

48. (a) 9.6 cm (b) 3 cm (c) $9\frac{3}{5}$ cm

49. $7\frac{1}{2}$ cm

50. $7\frac{1}{2}$ cm

51. (a) $4\frac{1}{5}$ cm (b) $7\frac{1}{2}$ cm

52. $13\frac{1}{5}$

53. $9\frac{3}{13}$

54. 18 cm

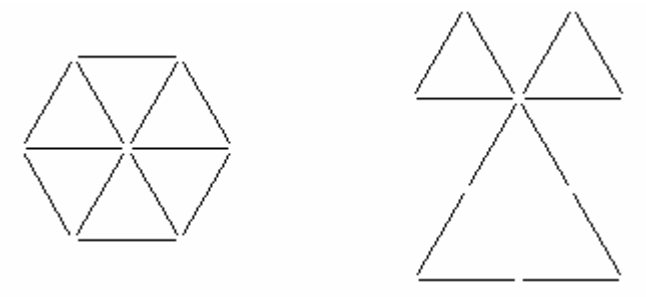
55. 7 cm

Chapter 9

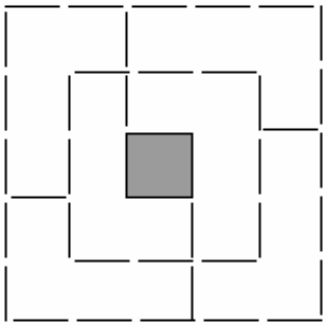
Secondary 3 Mathematics
Chapter 9 Area and Volume of Similar Figures and Solids

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 241)



Just For Fun (pg 241)



Just For Fun (pg 242)

$(\pi - 2) : 2$

Just For Fun (pg 244)

The diagonal joining the ends of the rectangle consists of two fine lines with a small gap in between forming an area of one square unit. Teachers can ask students to construct the figures using graph paper to let the students find out the answer.

Just For Fun (pg 251)

The dwarfs will have 10 times our body surface area and thus will lose heat and water 10 times our normal rate. They will die easily due to dehydration and heat loss.

The giants will be 1 000 times heavier than us but their surface area will only be 100 times larger . The ability of the legs to support their weights depends on the cross–sectional area of their legs. As the cross–sectional area of the legs will only be 100 times as large, they would not likely be able to support their heavier weights.

Do you notice that fat people have difficulty standing up for long? Have you noticed how thick the legs of an elephant are?

Secondary 3 Mathematics
Chapter 9 Area and Volume of Similar Figures and Solids

GENERAL NOTES

Very often a scale model of a building is made before the building is actually built. Some students may have visited places like mini-Siam in Bangkok, mini-China in Taiwan and so on during their vacations. They can discuss the models they saw in these places. In addition, they can bring some photographs of those places to show to the whole class. Teachers can also ask them if they know the scales used in making the models that they have seen.

Teachers can draw students' attention to commercial products which come in a range of sizes such as mineral water, toothpaste, hair shampoo, etc. Teachers can also ask them whether they are aware of the fact that the bottles or containers for each product are similar (different sizes but the same shape). Students can also find out whether the price is proportional to the net volume of the product.

Some students may possess toy cars which are miniature models of real cars. They can bring along some of them to classes especially those with inscriptions of the scales used in making the models. Using the scale factors available, teachers may want students to answer some hypothetical questions like: If the area of the windscreen of the model is x cm², what will be the area of the windscreen of the actual car? If the capacity of the petrol tank of a real car is y litres, what will be the capacity of the petrol tank of the model?

Teachers may also want to relate the present topic to the scales and map problems. Teachers can help students recall that if a map is drawn to a scale of $1 : n$, then the ratio of the area on the map to the corresponding area on the ground is $1 : n^2$.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: 35 min

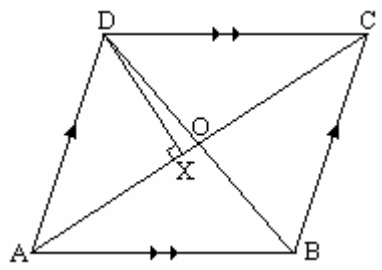
Class: _____ Marks: 12

Secondary 3 Multiple-Choice Questions

Chapter 9 Area and Volume of Similar Figures and Solids

1. In the diagram, ABCD is a parallelogram. Given that OC = 4 cm, $O\hat{X}D = 90°$ and the area of $\Delta BCD = 12cm^2$, the length of DX in cm is

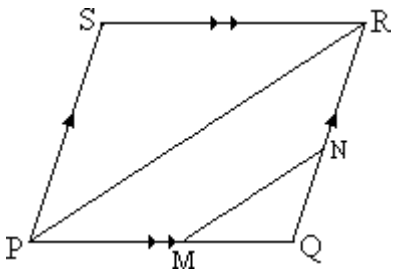
   (A) 4 (B)12
   (C) 8 (D) 3
   (E) 48



( )

2. In the diagram, PQRS is a parallelogram. Given that M and N are the mid–points of PQ and QR espectively, the value of $\frac{\text{Area of the } \Delta QMN}{\text{Area of PQRS}}$ is

   (A) $\frac{1}{4}$ (B) $\frac{1}{6}$
   (C) $\frac{1}{8}$ (D) $\frac{1}{10}$
   (E) $\frac{1}{12}$



( )

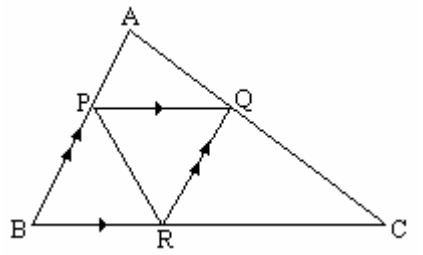
3. If the area of a face of a cube is increased by 44%, then its volume will be increased by

   (A) 44% (B) 173% (C) 73% (D) 199% (E) 299% ( )

4. The tents that Peter and Paul made are similar in shape. If Peter uses 9 times the amount of canvas as Paul, find the ratio of the volume of Peter's tent to Paul's tent.

   (A) 1:27 (B) 1:3 (C) 3:1 (D) 9:1 (E)27:1 ( )

5. In the diagram, PQ is parallel to BC, QR is parallel to AB and BR : RC = a : b. Find the ratio of the area of ΔPQR to the area of ΔABC.



(A) $a^2 : (a + b)^2$ (B) $ab : (a + b)^2$
(C) $ab : 2(a + b)^2$ (D) $2ab : (a + b)^2$
(E) $ab : (a^2 + b^2)$

( )

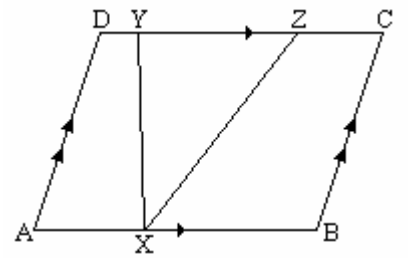
6. If a spherical balloon is inflated so that its diameter is tripled, then the volume is increased by a factor of

(A) 3 (B) 9 (C) 15 (D) 21 (E)27 ( )

7. If each edge of a cube is increased by 140%, the percentage increase in the surface area is

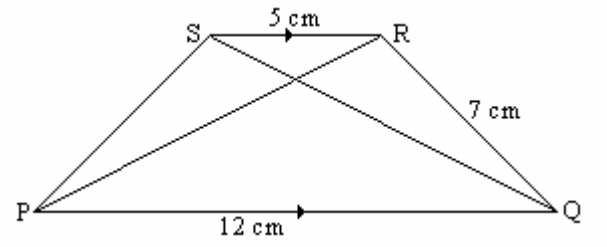
(A) 176 (B) 276 (C) 276 (D) 476 (E)576 ( )

8. In the diagram, ABCD is a parallelogram. If YZ = $\frac{1}{2}$AB, what fraction of the area of ABCD is the area of ΔXYZ?



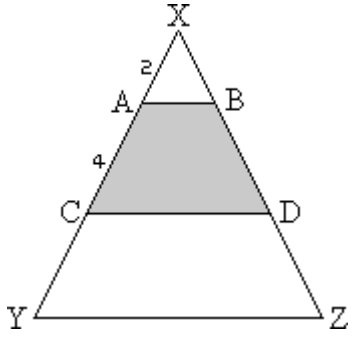
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$
(C) $\frac{1}{4}$ (D) $\frac{1}{5}$
(E) $\frac{1}{6}$ ( )

9. In the diagram, PQRS is a trapezium in which SR is parallel to PQ. Given that SR = 5 cm, RQ = 7 cm, PQ = 12 cm and the area of ΔPQR is 21 cm², the area of ΔSQR is



(A) 15 cm² (B) $8\frac{3}{4}$ cm²
(C) $12\frac{1}{4}$ cm² (D) $50\frac{2}{5}$ cm²
(E) 36 cm² ( )

10. In the diagram, AB // CD // YZ. If XA = 2 cm, AC = 4 cm, CY = 6cm and the area of ΔXYZ is 54 cm², then the area of the shaded region is

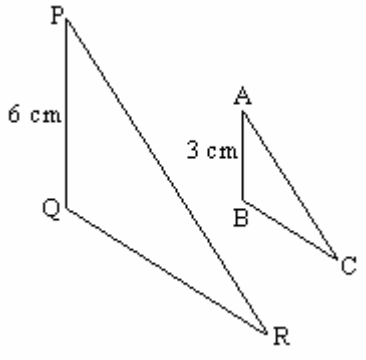


(A) 27 cm² (B) 18 cm²
(C) $20\frac{1}{4}$ cm² (D) 12 cm²
(E) 24 cm²

( )

11. In the diagram, triangles ABC and PQR are similar, PQ = 6 cm and AB = 3cm. If the area of ΔPQR is 40 cm², the area of ΔABC is

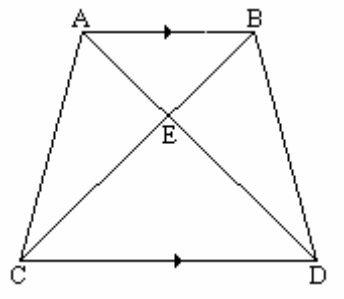
(A) 80 cm² (B) 18 cm²
(C) 8 cm² (D) 10 cm²
(E) 20 cm²



( )

12. In the diagram, ABCD is a trapezium in which AB : CD = 2 : 5. If the area of ΔCDE is 75 cm², then the area of ABCD is

(A) 100 cm² (B) 147 cm²
(C) 195 cm² (D) 300 cm²
(E) 317 cm²



( )

Answers

1. D	2. C	3. C	4. E	5. A	6. E
7. D	8. C	9. B	10. D	11. D	12. B

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: 

Secondary 3 Mathematics Test
Chapter 9 Area and Volume of Similar Figures and Solids

1. If a sphere of radius 1.5 cm weighs 135 g, find the weight in grams of a sphere with radius 2 cm. [3]

2. The lengths of the sides of three cubes are in the ratio 3:2:1.
   (a) If the total surface area of the smallest cube is 22 cm^2 , find the total surface area of the largest cube. [2]
   (b) If the volume of the largest sphere is 81 cm^3 , find the volume of the second largest cube. [2]

3. A bottle of height 8 cm has a volume of 120 cm^3 . Find the volume of a similar bottle of height 24 cm. [2]

4. John has two spheres. If the ratio of the radii of the two spheres is 2:3, find the ratio of their
   (a) surface areas, (b) volumes. [2]

5. The ratio of the surface areas of two similar cones is 0.72:3.92. Find the ratio of their
   (a) base diameters, (b) volumes. [3]

6. A model of a building is made to a scale of 1:300. The building has a volume of 5 400 m^3 . Calculate the volume of the model in cubic metres, giving your answer in standard form. [3]

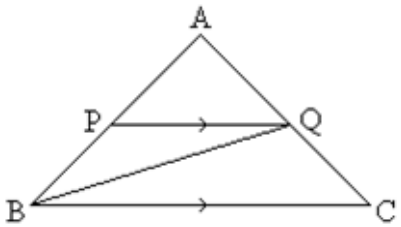
7. (a) A cylindrical tower is 200 m high, and its diameter is 20 m. Taking $\pi = 3.14$, find its volume. [2]
   (b) Peter is making a clay model of the tower with a height of 20 cm. Find the volume of the clay needed in m^3 . [3]

8. A small bottle is geometrically similar to a large one and the heights of the two bottles are 15 cm and 45 cm respectively. Write down
   (a) the ratio of the volume of the smaller bottle to that of the larger bottle,
   (b) the capacity of the larger bottle in litres if the capacity of the smaller bottle is 500 ml. [3]

9. A small traffic marker is geometrically similar to a large one and the diameters of the two markers are 12 cm and 24 cm respectively. Write down
   (a) the ratio of the total surface area of the small marker to that of the large marker,
   (b) the total surface area of the small marker if the total surface area of the large marker is 196 cm^2 . [3]

10. The area of two quadrilaterals are 49 cm^2 and 64 cm^2 . Find the ratio of their corresponding sides if they are similar. [2]

11. In the diagram, *PQ* is parallel to *BC*.
Given that *AQ* = 6 cm, *AC* = 15 cm
and the area of ΔAPQ = 16 cm^2 , find

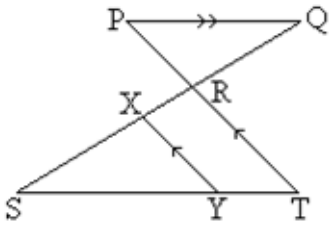


   (a) the area of ΔABC, [2]
   (b) the area of *PBCQ*, [1]
   (c) the area of ΔBCQ. [3]

12. A statue was melted down and recast into smaller, similar figures one-tenth of the original height. Given that a small figure is 30 cm tall and weighs 1.8 kg, find
   (a) the height of the original statue, [1]
   (b) the weight of the original statue. [2]

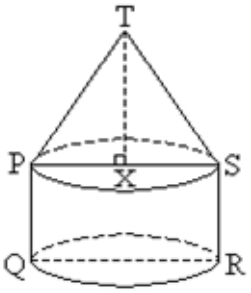
13. The surface area of two containers are in the ratio 144#:# 25. If the bigger container has a height of 96 cm and a volume of 5 184 cm^3 , calculate
   (a) the height of the smaller container, [2]
   (b) the volume of the smaller container. [2]

14. In the diagram, *PQ* // *ST*, *XY* // *RT*.



   (a) Prove that ΔPQR and ΔSXY are similar. [3]
   (b) Find the length of *PT* given that *QR* = 4 cm, *RX* = 2 cm, *XS* = 8 cm and *XY* = 16 cm. [3]
   (c) Find the numerical value of (i) $\frac{QR}{SR}$
   (ii) $\frac{area\ of\ \Delta PQR}{area\ of\ \Delta TSR}$ [2]
   (d) Given also that the area of ΔPQR is 24 cm^2 , calculate the area of ΔTSR. [1]

15. A building is made up of a cylinder and a cone on its top.
Given that TX = 5 m, XS = 12 m, TS = 13 m and PQ = 10 m,
find, taking $\pi = 3.14,2$
(a) the total surface area of the building, [3]
(b) the volume of the building, [3]
(c) the total surface area and the volume of a model of the building given that the diameter of the model is 120 cm. [4]



16.The model of a boat is 150 cm in length and the length of the actual boat is 18 m.
(a) If it costs \$2.50 to paint the model, what will it cost to paint the actual boat? [2]
(b) If the weight of the model is 3 kg, what is the weight of the actual boat if it is made of the same material? [2]

17. Two containers shown in the diagram are geometrically similar.

The height of the smaller container is 8 cm and the height of the larger container is 12 cm.
(a) The top of the smaller container has a circumference of 54 cm.
Find the circumference of the top of the smaller container. [2]
(b) Find the ratio of the area of the base of the smaller container to that of the larger container. [1]
(c) Both containers are filled with paint. The cost of the paint in the larger container is \$40.50.
Find the cost of the paint in the smaller container. [2]

18. The areas of the bases of the two similar glasses are in the ratio of 4#:#25.
(a) Find the ratio of the circumferences of the tops of the glasses. [1]
(b) Given that the capacity of the larger glass is 625 ml, find the capacity of the smaller glass. [2]

19. Two similar cones are such that the ratio of the areas of their circular bases is 36#:#4.
Find (a) the ratio of the diameters of the circular bases, [2]
(b) the ratio of the volumes of the cones. [1]

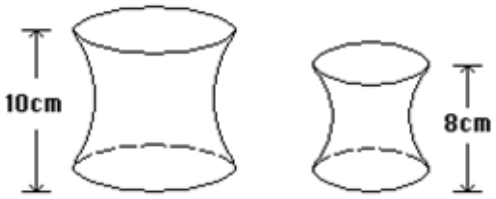
20. The volumes of two pyramids are 125 cm^3 and 216 cm^3 . Find the ratio of their corresponding heights if they are similar. [2]

21. In the diagram, *TU // PQ*, *SQ* = 3 cm, *RQ* = 9 cm, *PU* = 6 cm and *US* = 2 cm. Given that the area of ΔPRS is 32 cm^2 and *RT* = 2.4 cm, calculate
(a) the length of *TU*, [2]
(b) the area of ΔPQR, [2]
(c) the area of trapezium *RSUT*. [2]

22. The areas of the bases of two similar pyramids are in the ratio 9:25.
(a) Find the ratio of the heights of the pyramids. [1]
(b) Given that the volume of the larger pyramid is 275 cm^3 , find the volume of the smaller pyramid. [2]

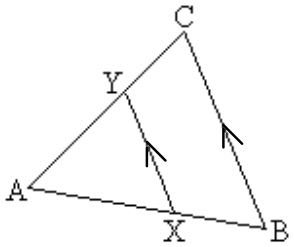
23. Two similar spheres have diameters of 20 cm and 28 cm respectively. The larger sphere has a volume of 1 715 cm^3 . Calculate the volume of the smaller sphere. [2]

24. The two containers shown are geometrically similar with respective heights of 8 cm and 10 cm.

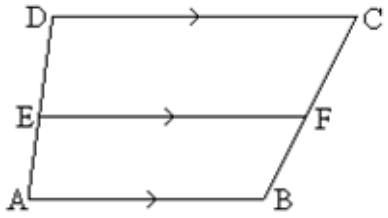


(a) The diameter of the base of the smaller container is 2.4 cm.
Calculate the diameter of the base of the larger container. [2]
(b) The containers are completely filled with flour. Given that the large container holds 1.25 kg, find the mass of the flour in the small container. [3]

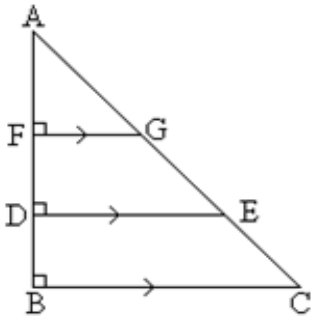
25. *ABC* is a triangle in which *XY // BC*.
Given that 5*AX* = 2*XB* and the area of ΔAXY = 16 cm^2 ,
find the area of the quadrilateral *XYCB*. [4]



26. The diagram shows a trapezium *ABCD* in which *AB // DC*. The area of trapezium *ABCD* is 21 cm^2 , *AB* = 2.5 cm, *CD* = 4.5 cm and *EF* = 4 cm respectively. Given that *EF // AB*, find the area of *ABFE*. [4]



27. The diagram consists of a right-angled ΔABC. *D* is the mid-point of *AB* and *F* is the mid-point of *AD*. Given that *AG // DE // BC* and the area of the triangle is 36 cm^2 , find the area of the trapezium *FGED*. [4]



28. A bucket with a height of 16 cm holds $4\frac{1}{2}$ litres. How many litres would a similar bucket hold if it had a height of 32 cm? [2]

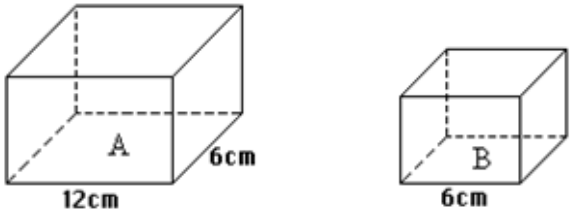
29. A statue is made from 4 050 cm^3 of metal.
    (a) Given that the density of the metal is 4 g/cm^3 , calculate the mass (in kg) of the statue. [2]
    (b) The statue is 57 cm high. An accurate scale model of the statue is made from 150 cm^3 of metal. Calculate the height (in cm) of the model. [2]

30. A model of a boat is made on the scale factor of $\frac{1}{5}$. If it costs \$1.60 to paint the hull of the model, what will it cost to paint the hull of the boat using the same paint? [3]

31. A certain brand of tea is sold in two sizes, similarly packed in tins of similar shape. The heights of the tins are 12 cm and 18 cm and the prices are \$7.20 and \$21.00. How much can you save in the purchase of the larger tin? [4]

32. A large ingot of metal is melted down and made into 125 small ingots all similar in shape to the original ingot. If the length of each small ingot is 5.2 cm, what was the length of the original ingot? [3]

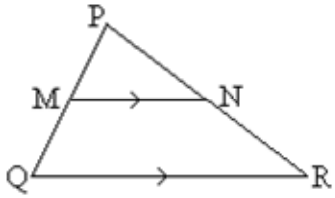
33. The diagram shows two similar cuboids.
(a) Find the volume of *B* if the volume of *A* is 128 cm³. [2]
(b) Find the ratio of the total surface area of *A* to that of *B*. [1]



34. Two solid spheres have surface areas of 6 cm^2 and 54 cm^2 respectively. The mass of the larger sphere is 81 kg. Find the mass of the smaller sphere. [3]

35. A container has a surface area of 2 000 cm^2 and a capacity of 8.75 litres. Find the surface area of a similar container which has a capacity of 15.12 litres. [3]

36. Given that *MN* = 4 cm, *QR* = 6 cm and the area of *MNRQ* = 20 cm^2, find the area of ΔPMN below. [3]



37. Two similar cylindrical containers have base radii of 9 cm and 12 cm respectively. If the capacity of the smaller container is 297 cm^3, find the capacity of the larger container. [3]

38. The masses of two similar solids are 32 kg and 108 kg respectively. If the surface area of the smaller solid is 576 cm^2, find the surface area of the larger solid. [3]

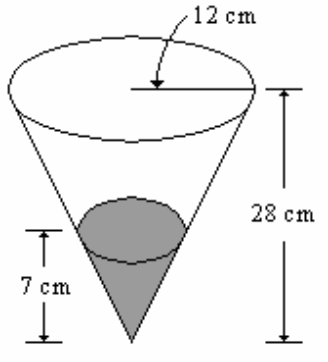
39. A cylindrical container has a circumference of 45 cm and a capacity of $6\frac{1}{4}$ litres. Find the capacity of a similar cylinder of circumference 36 cm. [3]

40. Two solid cones have curved surface areas of 8 m^2 and 128 m^2 respectively and the mass of the smaller one is 12 kg. Find the mass of the larger cone. [3]

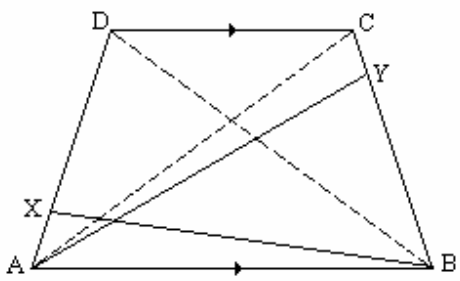
41. Three triangles T_1, T_2 and T_3 are all similar to one another. The ratio of the lengths of a pair of corresponding sides in T_1 and T_2 is 2 : 5. The ratio of the lengths of a pair of corresponding sides in T_2 and T_3 is 10 : 9. Find the ratio of the areas of triangles T_1 and T_3. [4]

42. If the diameter of a soap bubble increases by 30%, what is the percentage increase in its volume? [3]

43. The diagram shows a container in the form of an inverted cone with a base radius of 12 cm and a height of 28 cm.
(a) Find the capacity of the container, taking π to be $\frac{22}{7}$.
(b) If water is poured into the container and the depth of water is 7 cm, find the volume of the water. [5]



44. In the diagram, AB and CD are the parallel sides of a trapezium ABCD. X is a point on AD such that AD = 5AX and Y is a point on BC = 5YC.
(a) What is the value of the ratio $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ABY}$?
(b) Find the ratio of the area of ΔABX and ΔABD.
(c) If the area of ΔABY is 72 cm², find the area of ΔABX. [7]

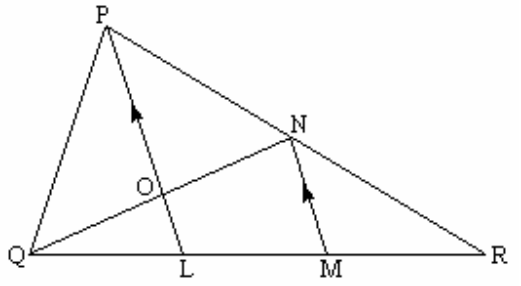


45. The volume of a cube is 64 times that of another cube.
(a) If the side of the smaller cube is 5 cm, find the side of the larger cube.
(b) If the total surface area of the larger cube is 528 cm², find the total surface area of the smaller cube. [4]

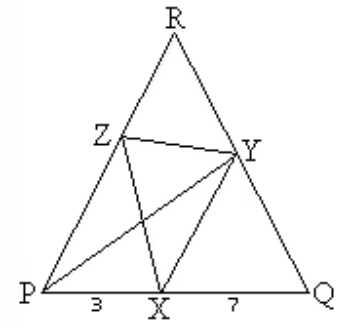
46. One side of a polygon of area 72 cm² is 6 cm. Find the area of a similar polygon in which the corresponding side is 2 cm. [3]

47. A prism has a volume of 80 cm³. What is the volume of a similar prism whose height is 5 times the height of the given prism? [3]

48. In the diagram, N is the mid-point of PR, L is a point on QR such that QR = 3QL and MN is parallel to LP. Find the ratio of the area of ΔOQL to the area of quadrilateral OLRN. [3]



49. In the diagram, X, Y and Z are points on sides PQ, QR and RP of ΔPQR respectively.
    (a) Given that ΔPQY and quadrilateral XQYZ have equal areas,
        (I) show that ΔPXY and ΔXYZ have equal areas.
        (ii) explain why XY is parallel to PR.
    (b) If it is given further that PX = 3 cm, XQ = 7 cm and the area of ΔPQR is 20 cm², find the area of quadrilateral XQYZ. [6]



50. Two cones have base radii 12 cm and 18 cm, and vertical heights 22 cm and 33 cm.
    (a) If the curved surface area of the smaller cone is $\alpha\pi\text{cm}^2$, find the slant height of the larger cone in terms of α.
    (b) If the volume of the larger cone is V cm³, express the volume of the smaller cone in terms of V. [4]

51. Two similar cones have diameter 15 cm and 12 cm respectively. If the total surface area of the first cone is 300 cm², find the total surface area of the second cone. [3]

52. Two similar glasses have heights 9 cm and 12 cm respectively. If the capacity of the smaller glass is 360 cm², find the capacity of the bigger glass. [3]

53. A marble statue of height 3 m weighs 54 kg. What is the weight of a similar marble statue if its height is 2 m? [3]

54. Two similar biscuit tins have volumes of 6 400 cm³ and 2 700 cm³. If the height of the first tin is 14 cm, find the height of the second tin. [3]

55. Two similar cups have volumes of 200 cm³ and 1 600 cm³. If the height of the second cup is 12 cm, find the height of the first cup. [3]

56. Two similar cakes of diameters 15 cm and 20 cm are sold at $12.00 and $26.00 respectively. Find out which cake is cheaper and by how much, to the nearest 10 cents. [3]

57. A model of a cargo–ship is made on a scale of 1:30. If the volume of the model is 2 m^3, what is the volume of the actual ship? [3]

58. Two watering cans are of the same shape. The smaller one is 45 cm high and can hold $4\frac{1}{2}$ litres of water. If the larger one is 60 cm high, how much water can it hold? [3]

Answers

1. 320 g

2. (a) 198 cm^2 (b) 24 cm^3

3. 3 240 cm^3

4. (a) 4:9 (b) 8:27

5. (a) 3:7 (b) 27:343

6. $2 \times 10^{-4} m^3$

7. (a) 62 800 m^3 (b) 62.8 m^3

8. (a)1:27 (b) 13.5 litres

9. (a) 1:4 (b) 49 cm^2

10. 7:8

11. (a) 100 cm^2 (b) 84 cm^2 (c) 60 cm^2

12. (a) 3 m (b) 1 800 kg

13. (a) 40 cm (b) 375 cm^3

14. (b) 17.92 cm
    (c) (i) $\frac{2}{5}$ (ii) $\frac{4}{25}$
    (d) 150 cm^2

15. (a) 1 244.23 m^2
    (b) 5 278.56 m^3
    (c) 17.92 m^2 , 9.12 m^3

16. (a) \$4 320 (b) 5 184 kg

17. (a) 36 cm (b) 4:9 (c) \$12

18. (a) 2:5 (b) 40 ml

19. (a) 3:1 (b) 27:1

20. 5:6

21. (a) $4\frac{1}{2}$ cm (b) 48 cm^2 (c) 14 cm^2

22. (a) 3:5 (b) 59.4 cm^3

23. 625 cm^3

24. (a) 3 cm (b) 640 g

25. 180 cm^2

26. $14\frac{5}{8}$ cm^2

27. 16.75 cm^2

28. 36 litres

29. (a) 16.2 kg (b) 19 cm

30. \$200

31. $3.30

32. 26 cm

33. (a) 16 cm^3 (b) 4:1

34. 3 kg

35. 2 880 cm^2

36. 16 cm^2

37. 704 cm^3

38. 1 296 cm^2

39. $3\frac{1}{5}$ litres

40. 768 kg

41. 4:9

42. 119.7%

43. (a) 4 224 cm^3 (b)66 cm^3

44. (a) $\frac{5}{4}$ (b) $\frac{1}{5}$ (c) 18 cm^2

45. (a) 20 cm (b) 33 cm^2

46. 8 cm^2

47. 10 000 cm^3

48. $\frac{1}{5}$

49. (b) 14cm^2

50. (a) $\frac{3a}{44}$ cm (b) $\frac{8}{27}V$ cm³

51. 129 cm²

52. $853\frac{1}{3}$ cm³

53. 16 kg

54. $10\frac{1}{2}$ cm

55. 6 cm

56. The second cake and it is cheaper by $2.40.

57. 54 000 m³

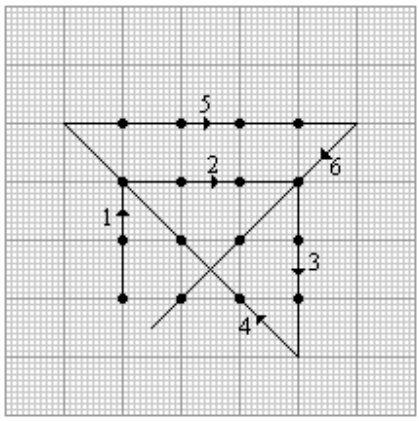
58. $10\frac{2}{3}$ litres

Chapter 10

Secondary 3 Mathematics
Chapter 10 Trigonometrical Ratios

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 272)



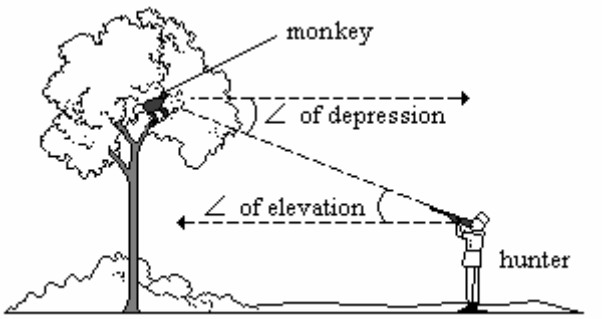
Secondary 3 Mathematics
Chapter 10 Trigonometrical Ratios

GENERAL NOTES

Teachers may wish to introduce one trigonometrical ratio at a time especially for weaker students. For brighter students, all three trigonometrical ratios can be introduced at one time. To help students to memorise these trigonometrical ratios more easily, teachers may wish to introduce the mnemonics, TOA-CAH-SOH which can be made to sound like "big–foot lady " in the Fujian dialect. Plenty of practice should be given to identify the opposite, adjacent and hypotenuse sides with reference to a given angle as many students may find these confusing at the initial stage.

When using the calculator, students must always be reminded to check that the mode is in 'degree'.

A mnemonic for the angle of elevation and the angle of depression is shown in the following story:



Draw on the board or a transparency the picture of a monkey playing in a tree and a weary hunter with a gun.

The hunter has been looking for animals without much success for many hours. Suddenly he hears the noise from a monkey up in a tree in front of him. His gaze that was straight initially is now raised through an angle to spot the monkey. He is *elated* by the sight of the monkey (hence, the angle of elevation).

The monkey, which is busily looking ahead for a mate of the fairer sex, hears the click of the gun. He looks down at an angle and is very *depressed* to see the hunter's gun (hence, the angle of depression).

The Geometers' Sketchpad (GSP) can be used to illustrate the various trigonometrical ratios.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: 

Secondary 3 Mathematics Test
Chapter 10 Trigonometrical Ratios

1. Given that $\angle A$ is an acute angle such that $\cos A = \frac{12}{13}$, construct $\angle A$ and measure its value correct to the nearest degree. [3]

2. Given that $\sin \theta = \frac{5}{13}$ and that θ is an acute angle, write down the values of
   (a) $\cos \theta$ (b) $\tan (90 - \theta)$ [4]

3. Given that $\cos \theta = \frac{3}{5}$ where θ is acute, find the numerical value of $2 \sin \theta + 5 \tan \theta$, giving your answer as a fraction in its lowest terms. [4]

4. Given that $\sin \theta = \frac{12}{13}$ and θ is an acute angle, find the numerical values of
   (a) $2 \cos \theta$ (b) $3 \tan \theta$ [3]

5. If $\sin \theta = \frac{12}{13}$ where θ is acute, find the values of (a) $\cos \theta$ [2]
   (b) $\tan (90 - \theta)$ [2]

6. If $\sin \theta = \frac{7}{25}$ where θ is acute, find the value of $2 \cos \theta + 3 \tan (90 - \theta)$. [4]

7. Given that $\tan x = \frac{5}{12}$ where x is an acute angle, find the values of
   (a) $2 \sin x$, (b) $3 \cos x - \sin x + \tan x$. [4]

8. If $\sin x = \frac{15}{17}$ and x is acute, find the value of each of the following, giving your answer as a fraction in its lowest terms.
   (a) $\tan x + 2 \cos x$ [2]
   (b) $3 \cos x - 2 \sin x$ [2]

9. If $\sin\theta = \frac{40}{41}$ where θ is acute, find the value of $2\cos\theta + 3\tan(90 - \theta)$, giving your answers as a fraction in its simplest form. [4]

10. If $\tan\theta = 2\frac{2}{5}$ where θ is acute, find the values of (a) $2\sin\theta$ [2]

(b) $\cos(90 - \theta)$ [1]

11. If $\tan\theta = \frac{a}{b}$, where θ is acute and a and b are positive, find, in terms of a and b, the values of (a) $\sin\theta$ [2]

(b) $\cos\theta$ [2]

12. If $\cos x = \frac{8}{17}$ where x is acute, find the value of each of the following, giving your answer as a fraction in its lowest terms.

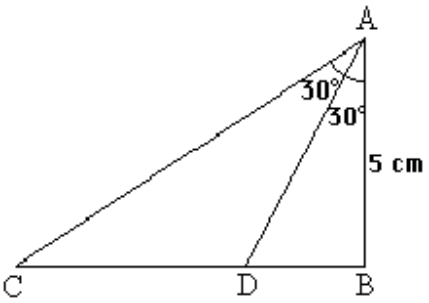
(a) $2\tan x + 3\sin x$ (b) $3\sin x - 4\tan x$ [4]

13. In the diagram, AB = 5 cm, $\angle ABC = 90°$ and $\angle BAD = \angle CAD = 30°$. Using as much of the information below as possible, calculate

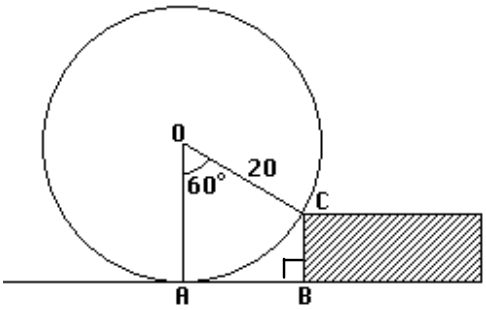
(a) CD [2]

(b) AC [1]

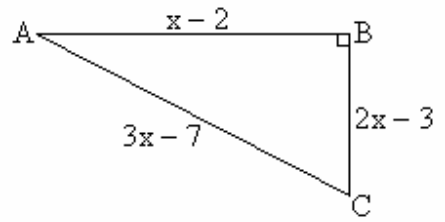
[Given that $\sin 30° = 0.5 = \cos 60°$,
$\sin 60° = \cos 30° = 0.87$,
$\tan 30° = 0.58$ and $\tan 60° = 1.73$.]



14. The diagram shows a wheel of radius 20 cm in contact with the horizontal ground at *A* and touching the vertical step at *C*. Calculate
(a) *AB* [2]
(b) *BC* [2]
[Given that sin 60° = 0.87, cos 60° = 0.5, tan 60° = 1.73.]



15. In the figure, ∠*ABC* = 90°, *AB* = (x – 2)cm, *BC* = (2x – 3)cm and AC = (3x – 7)cm.
Calculate (a) the value of *x*, [3]
(b) ∠BAC [2]



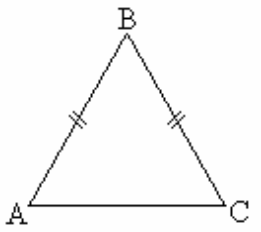
16. The lengths of the two diagonals of a rhombus are 32 cm and 16 cm respectively. Calculate the length of a side of the rhombus, giving your answer correct to 2 decimal places. [4]

17. The angle of depression of a boat 58.5 m from the base of a cliff is 35.6°. How high is the cliff? (Give your answer correct to 1 decimal place.) [3]

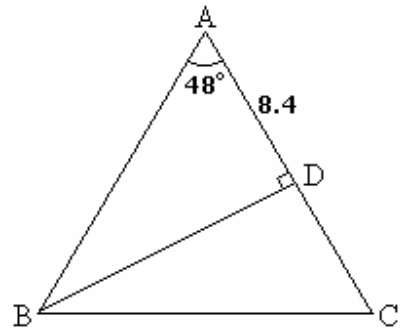
18. From a window 45 m high in a building, the angle of depression of a bus–stop is 36.7°. Calculate the distance of the bus–stop from the foot of the building. [3]

19. In Δ*ABC*, *AB* = 14.6 cm, ∠ACB= 48.9° and the length of the perpendicular from *A* to *BC* is 9.4 cm. Calculate the area of Δ*ABC*. [5]

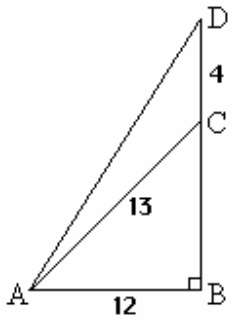
20. In ΔABC, $AB = BC = 15.4$ cm and $\angle ABC = 54°$. Calculate the area of ΔABC. [2]



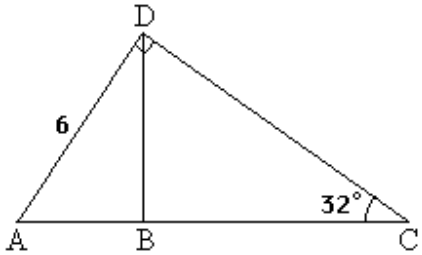
21. In the figure, $AB = AC$, $\angle ADB = 90°$, $\angle BAC = 48°$ and $AD = 8.4$ cm. Calculate (a) BD,
(b) CD. [4]



22. In the figure, $\angle ABD = 90°$, $AB = 12$ cm, $AC = 13$ cm and $CD = 4$ cm. Calculate
(a) AD, [3]
(b) $\angle ACB$ [2]
(c) $\angle CAD$ [3]

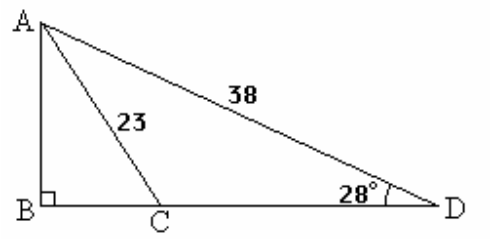


23. In the diagram, $\angle ADC = \angle ABD = 90°$, $AD = 6$ cm and $\angle BCD = 32°$.
Calculate (a) BD [2]
(b) AC [3]
(c) CD [2]

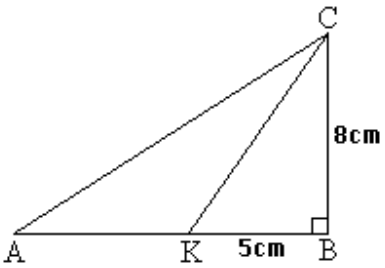


24. The sides of a right–angled triangle are $2x$ cm, $(3x – 1)$ cm and $(3x + 1)$cm.
(a) Form an equation in x and solve it to find x. [4]
(b) With the value of x, find the perimeter and the area of the triangle. [3]

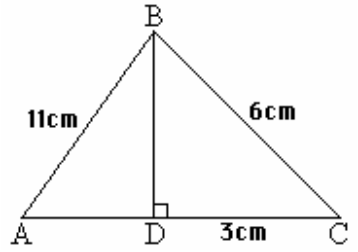
27. In the figure, ∠ABC = 90°, ∠ADB = 28°, AC = 23 cm and AD = 38 cm. Calculate ∠BAC. [4]



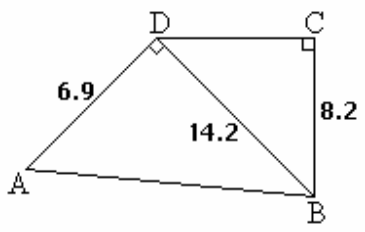
28. In the diagram, ∠ABC = 90°, BC = 8 cm and KB = 5 cm.
(a) Calculate the length of CK, giving your answer correct to 2 decimal places. [2]
(b) Write down the numerical value of tan∠CKB. [1]
(c) Given that 2AK = 3KB, calculate the size of ∠CAB and the length of AC. [5]



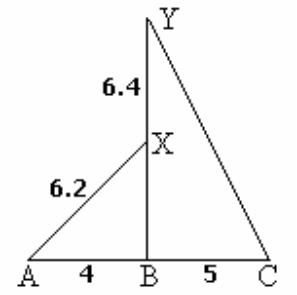
29. In the diagram, AB = 11 cm, BC = 6 cm, CD = 3 cm and BDC = 90°. Calculate
(a) BD [2]
(b) AC [2]
(c)∠ABC [2]
(d) the area of Δ ABC [2]



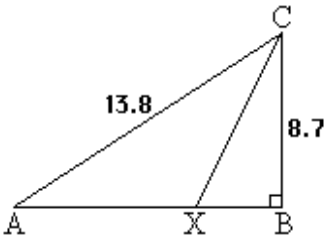
30. In the figure, ∠BCD= ∠ADB = 90°, AD = 6.9 cm, BD = 14.2 cm and BC = 8.2 cm. Calculate∠ABC. [4]



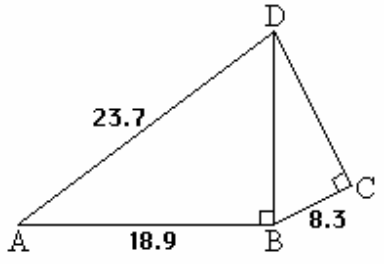
31. In the figure, ∠ABX = 90°, AB = 4 cm, BC = 5 cm, AX = 6.2 cm and XY = 6.4 cm.
Calculate (a) CY, [3]
(b). ∠BCY [3]



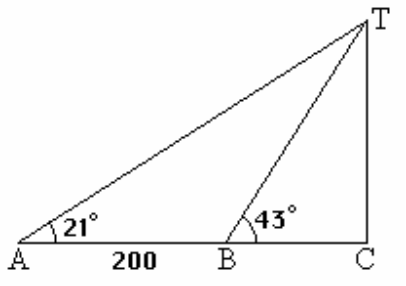
32. In the figure, ∠ABC = 90°, BC = 8.7cm, AC = 13.8 cm and X is the mid–point of AB. Calculate the length of CX , giving your answer correct to 1 decimal place. [4]



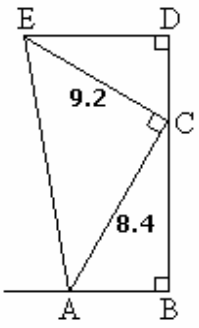
33. In the figure,∠ABD =∠BCD = 90°, AB = 18.9 cm, AD = 23.7 cm and BC = 8.3 cm. Calculate the length of CD, giving your answer correct to 2 decimal places. [4]



34. At the point A, the angle of elevation of a ship from the top of a cliff is 21°. After sailing for 200 m towards the cliff, the angle of elevation becomes 43°. Calculate the height of the cliff. [6]



35. In the figure, $\angle ABC = \angle ACE = \angle CDE = 90°$. AC = 8.4 cm, CE = 9.2 cm, and $\angle BAC = 52°$.
Calculate (a) BC, [2]
(b) DE, [3]
(c). $\angle AEC$ [2]



36. The angle of elevation of the top of a building from a point on the ground 220 m away from the foot of the building is 16°. What is the height of the building? [3]

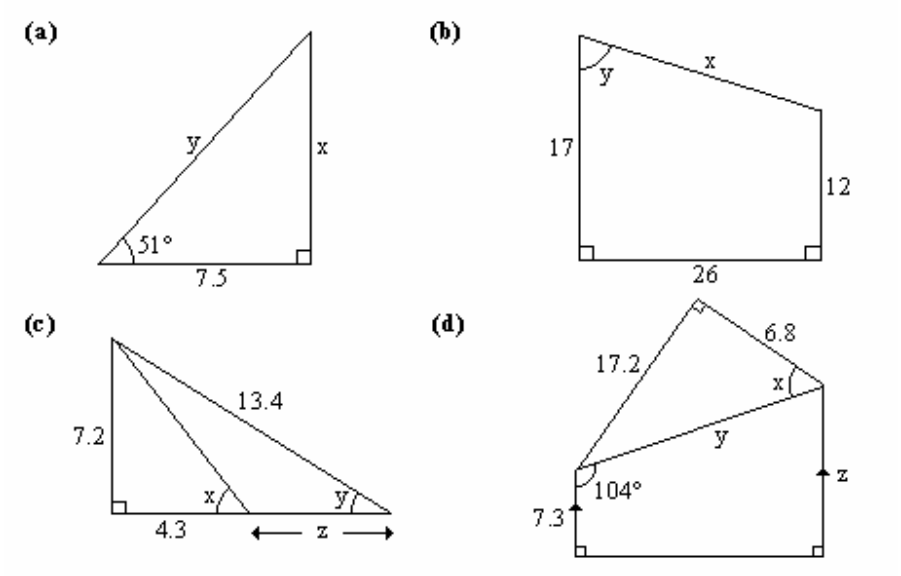
37. A 5.2 m long ladder rests against the side of a house such that its foot is 1.7 m from the foot of the house. Find the angle at which the ladder makes with the horizontal. [3]

38. The angle of depression of a small boat from the top of a mast of a cruise ship is 9.8°. If the top of the mast is 76 m above the water level, find the distance from the boat to the top of the mast. [3]

39. A person, standing on the bank of a river, observes that the angle of elevation of the top of a tree on the opposite bank to be 48°. When he walks 30 m away from the bank, he finds the angle of elevation of the top of the tree to be 36°. Find the height of the tree and the width of the river. [5]

40. A boy standing in the middle of two flag poles 78 m apart finds that the angle of elevation of the tops of the flag poles from the point where he is standing are 34° and 43°. Find the difference in the heights of the two flagpoles. [4]

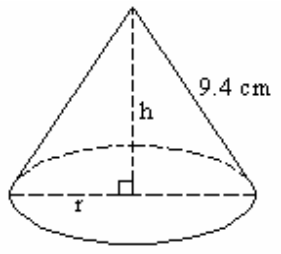
41. Find the unknown sides (in cm) and angles indicated in each of the following figures: [12]



42. Find the angle of elevation of the top of a mast that is 35 m high from a point 28 m away from its foot on level ground. [3]

43. If $\sin x = \frac{12}{13}$, find the value of each of the following, giving your answer as a fraction in its lowest terms.
    (a) $2 \cos x + 3 \sin x$
    (b) $3 \tan x - \cos x$
    (c) $5 \sin x - 4 \tan x$
    (d) $2 \sin x - \cos x + \tan x$ [5]

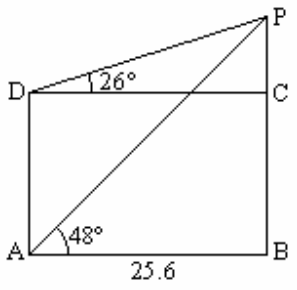
44. The slant height of a right–circular cone measures 9.4 cm and the angle at the vertex is 68°. Calculate
    (a) the height of the cone,
    (b) the radius of the base. [6]



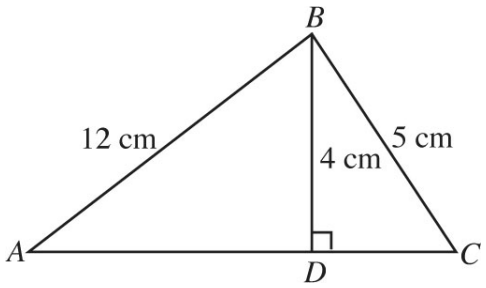
45. From the top of a block of flats, the angle of depression of a car 45 m from the foot of the flat is 63°. Calculate the height of the flat. [3]

46. A tower 48 m high casts shadow 55 m long. Find the angle of elevation of the sun. [3]

47. In the figure ABCD is a rectangle in which AB = 25.6 cm, ∠PAB = 48° and ∠PDC = 26°. Calculate
(a) PC, (b) PD,
(c) area of ABCD. [7]

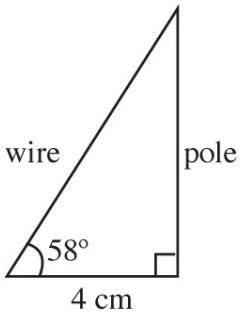


48. In the figure, *AB* = 12 cm, *BC* = 5 cm, *BD* = 4 cm and *BDC* = 90°. Calculate at the length of *AC* and the angle *ABC*. [5]

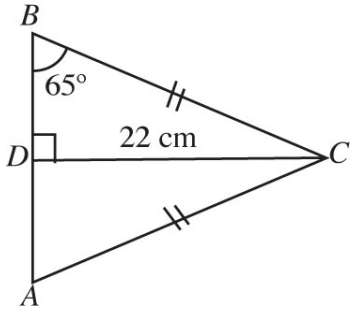


49. The angle subtended at the centre of a circle by a chord of length, 18 cm is 120. Find the distance between the chord and the centre. [3]

50. Find the length of the wire supporting a pole in the given figure. [3]



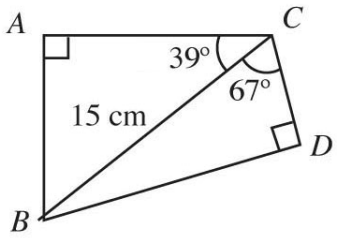
51. In the figure, *AC* = *BC*, *CD* = 22 cm and ∠*ABC* = 65°. Calculate the perimeter and area of triangle *ABC*. [6]



52. Calculate the hypotenuse of a right-angled triangle whose other sides are 6.2 cm and 8.4 cm. Also calculate the values of the other two angles. [6]

53. If the altitude of an isosceles triangle is 40 cm and the length of its base is 30 cm, find the vertical angle and the perimeter. [4]

54. For the given figure, calculate (a) *AC*, (b) *BD*. [5]



55. From a point 20 m away from the foot of a building, the angles of elevation of the top and bottom of a flagpole which stands on the top of the building are 44° and 36° respectively. Find the height of the flagpole. [6]

Answers

1. 23º

2. (a) 12/13

   (b) 2 2/5

3. 8 4/15

4. (a) 10/13

   (b) 7 1/5

5. (a) 5/13

   (b) 5/12

6. 12 36/175

7. (a) 10/13

   (b) 2 125/156

8. (a) 2 111/136

   (b) –6/17

9. 1 187/1 640

10. (a) 1 11/3

    (b) 12/13

11. (a) $\frac{a}{\sqrt{a^2 + b^2}}$

    (b) $\frac{b}{\sqrt{a^2 + b^2}}$

12. (a) 6 27/68

    (b) –4 29/34

13. (a) 29° (b) 240 thousand tonnes

14. (a) 10cm
    (b) 17.3cm

15. (a) 4 ½

    (b) 67.4º

16. 17.89cm

17. 41.9m

18. 60.4m

19. 81.55cm^2

20. 95.9cm^2

21. (a) 9.33cm

    (b) 4.15cm

22. (a) 15cm
    (b) 67.4º
    (c) 14.3º

23. (a) 5.09cm
    (b) 11.3cm
    (c) 9.60cm

24. (a) $(2x)2 + (3x - 1)2 = (3x + 1)2 =, x = 3$
    (b) 24cm, 24cm^2

25. (-2, 2)

26. (a) 14.25cm
    (b) 52.3º
    (c) 22.74cm

27. 39.1º

28. (a) 9.43cm

    (b) 1 3/5

    (c) 32.6º

    (d) 14.84cm

29. (a) $\sqrt{27} = 5.20$cm
    (b) 12.7cm
    (c) 91.8º
    (d) 33cm^2

30. 80.6º

31. (a) 12.2cm
    (b) 65.8º

32. (a) 10.2cm

33. 11.64cm

34. 130.5m

35. (a) 6.62cm
(b) 7.25cm
(c) 42.4°

36. 63.1°

37. 70.9°

38. 446.5 m

39. Ht of tree = 63.0 m, width of river = 56.8 m

40. 10.06 m.

41. (a) x = 9.26 cm, y = 11.92 cm (b) x = 26.48 cm, y = 79.1°

(c) x = 59.2°, y = 32.5°, z = 7.0 cm (d) x = 68.4°, y = 18.5 cm, z = 11.8 cm

42. 51.3°

43. (a) $3\frac{7}{13}$ (b) $6\frac{53}{65}$ (c) $-4\frac{64}{65}$ (d) $3\frac{56}{65}$

44. (a) 7.79 cm (b) 5.26 cm

45. 88.3 m

46. 41.1°

47. (a) 12.5 cm (b) 28.5 cm (c) 408 cm²

48. 14.3 cm, 107.4°

49. 5.2 cm

50. 7.55 m

51. 69.1 cm, 225.7 cm^2

52. 10.44 cm, 36.4°, 53.6°

53. 41.1^{o}, 115.4 cm

54. (a) 11.66 cm (b) 13.81 cm

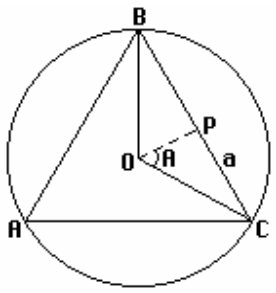
55. 4.78 m

Chapter 11

Secondary 3 Mathematics
Chapter 11 Further Trigonometry

ANSWERS FOR ENRICHMENT ACTIVITIES

Exploration (pg 308)



Join B to O and C to O. Join O to P, the mid-point of BC. Let R be the radius of the circle.

$B\hat{O}C = 2A$, $\therefore P\hat{O}C = A$ and $PC = \frac{1}{2}a$.

In ΔPOC, $\sin A = \frac{PC}{OC} = \frac{\frac{1}{2}a}{R} = \frac{a}{2R}$

$\therefore 2R = \frac{a}{\sin A}$

Similarly, by joining A to O and using the same reasoning,

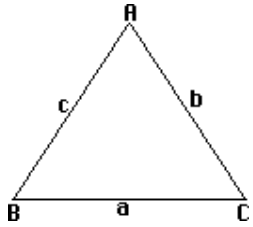
we can prove that $2R = \frac{b}{\sin B} = \frac{c}{\sin C}$

Just For Fun (pg 321)

These 3 figures are impossible to construct in real life.

Exploration (pg 304)

Heron's Formula for the area of a triangle



Consider ΔABC with sides a, b, and c.
Let $2s$ be the perimeter of ΔABC, hence $2s = a + b + c$.
s is then known as the semi-perimeter of the triangle.

Applying the Cosine Rule,

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

and using the identity $\sin^2 A + \cos^2 A = 1$

$\therefore \sin^2 A = 1 - \cos^2 A$

$= (1 + \cos A)(1 - \cos A)$

Substituting for $\cos A$,

$$\sin^2 A = \{ 1 + \frac{b^2 + c^2 - a^2}{2bc} \} \{ 1 - \frac{b^2 + c^2 - a^2}{2bc} \}$$

$$= \{ \frac{2bc + b^2 + c^2 - a^2}{2bc} \} \{ \frac{2bc - b^2 - c^2 + a^2}{2bc} \}$$

$$= \{ \frac{b^2 + 2bc + c^2 - a^2}{2bc} \} \{ \frac{-(b^2 - 2bc + c^2) + a^2}{2bc} \}$$

$$= \{ \frac{(b + c)^2 - a^2}{2bc} \} \{ \frac{-(b - c)^2 + a^2}{2bc} \}$$

$= \{ \frac{(b + c + a)(b + c - a)}{2bc} \} \{ \frac{(a + b - c)(a - b + c)}{2bc} \}$

Now $a + b + c = 2s$ (perimeter of ΔABC)

Hence $b + c - a = a + b + c - 2a = 2s - 2a$

$a + b - c = a + b + c - 2c = 2s - 2c$

$a - b + c = a + b + c - 2b = 2s - 2b$

Substituting into $\sin^2 A$, we have

$\sin^2 A = \frac{2s \cdot 2(s - a) \cdot 2(s - c) \cdot 2(s - b)}{4b^2c^2}$

$\therefore \quad \sin A = \pm \sqrt{\frac{4s(s - a)\,(s - b)\,(s - c)}{b^2c^2}}$

$\sin A = \frac{2}{bc}\sqrt{s(s - a)\,(s - b)\,(s - c)}$

i.e. $\frac{1}{2}\text{bc} \sin A = \sqrt{s(s - a)\,(s - b)\,(s - c)}$

But area of $\Delta ABC = \frac{1}{2}bc \sin A$

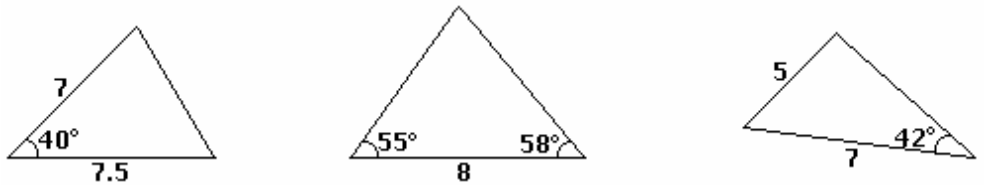
$\therefore$ Area of $\Delta ABC = \sqrt{s(s - a)\,(s - b)\,(s - c)}$

Secondary 3 Mathematics
Chapter 11 Further Trigonometry

GENERAL NOTES

The trigonometrical ratios of obtuse angles are important in this course. To help pupils have a better understanding of the concepts, teachers should derive the formulas for the sine and cosine rule. Pupils are also encouraged to use the Geometer's Sketchpad (GSP) to derive the sine rule using the Thinking Skill: Inferring. For pupils doing Additional Mathematics, this topic can be taught concurrently, covering angles from 0^o to 360^o.

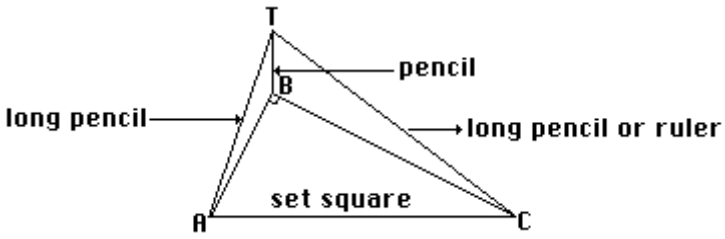
Pupils sometimes find difficulty in deciding whether to use the sine or cosine rule to solve a triangle. Plenty of practice should be given to the pupils to help them decide which rule would be most suitable to solve triangles. The triangles below may be used for the purpose.



(i) **(ii)** **(iii)**

To help pupils gain a better perspective of 3-D problems, wire models with strings or rods are useful tools to illustrate the triangles and angles involved. 2-D diagrams are unable to illustrate 3-D shapes effectively, e.g. the square faces of a cube. Thus it will be helpful to draw separate 2-D diagrams showing exact shapes of triangles and angles where they are needed.

The use of available materials such as set-squares, ruler, pencils, etc. to set up 3-D diagrams for better understanding is helpful to pupils especially during examination times.



The above diagram shows how a set square and a few pencils are used to show a ΔABC right-angled at B and a vertical post BT standing at B.

Common Errors

Pupils should be reminded to set the mode of their calculators to "Degree". Pointing out that use of angles in radians will be covered in the next chapter. Many pupils also fail to remember the formula correctly during test or examination time. One way to correct this mistake would be to insist that they write down the formula for each sum that they do.

XYZ SECONDARY SCHOOL

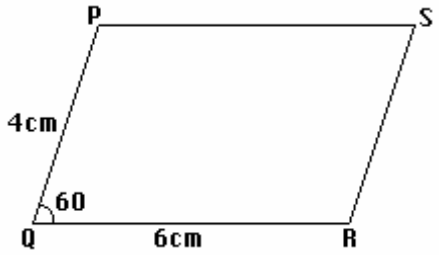
Name: _____ ( ) Date: _____

Time allowed: 35 min

Class: _____ Marks: 18

Secondary 3 Multiple-Choice Questions
Chapter 11 Further Trigonometry

1. *PQRS* is a parallelogram where $PQ = 4$ cm, $QR = 6$ cm and $P\hat{Q}R = 60°$. Find the area of the parallelogram in cm^2.



(A) $6\sqrt{3}$ (B) 12
(C) $12\sqrt{3}$ (D) 20
(E) 24

( )

2. The value of PR^2 in the diagram above is

(A) 24 (B) 28 (C) 52 (D) 76 (E) 82

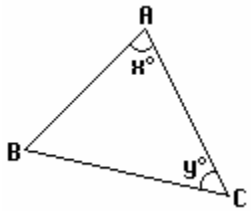
( )

3. The length of the sides of a triangle are 8 cm, 12 cm and 15 cm. The triangle must be

(A) isosceles (B) acute angled (C) right-angled
(D) obtuse angled (E) equilateral

( )

4. In the figure, $AB = AC$ and $BC > AB$. State which of the following is true.



I. $x > 60$ II. $y < x$ III. $y > 60$

(A) I only (B) II only
(C) III only (D) I and II only
(E) II and III only

( )

5. If x is acute and $\tan x = \frac{1}{\sqrt{2}}$, then the value of $\cos x$ is

(A) $\sqrt{\frac{1}{3}}$ (B) $\sqrt{\frac{2}{3}}$ (C) $\sqrt{\frac{3}{2}}$ (D) $\sqrt{3}$ (E) $\sqrt{\frac{1}{5}}$ ( )

6. In ΔPQR, $P\hat{R}Q = 57°$, $QR = 6.2$ cm and $PR = 7.8$ cm. To find the length of PQ, we can use the

(A) sine rule (B) cosine rule (C) Pythagoras' Theorem
(D) tangent ratio (E) area formula ( )

7. If $45° < x < 90°$, which of the following is true?

(A) $\tan x < \cos x < \sin x$ (B) $\cos x < \tan x < \sin x$
(C) $\sin x < \cos x < \tan x$ (D) $\cos x < \sin x < \tan x$
(E) $\sin x < \tan x < \cos x$ ( )

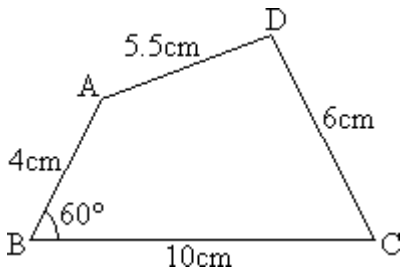
8. In ΔABC, $AB = 17$ cm, $AC = 8$ cm and $BC = 15$ cm. The area of ΔABC is

(A) 15 cm^2 (B) 30 cm^2 (C) 68 cm^2 (D) 120 cm^2 (E) 60 cm^2 ( )

9. In ΔPQR, $P\hat{Q}R = 30°$, $PQ = 8$ cm and $QR = 10$ cm. The area of ΔPQR is

(A) 10 cm^2 (B) $10\sqrt{3}$ cm^2 (C) 20 cm^2 (D) 30 cm^2
(E) $20\sqrt{3}$ cm^2 ( )

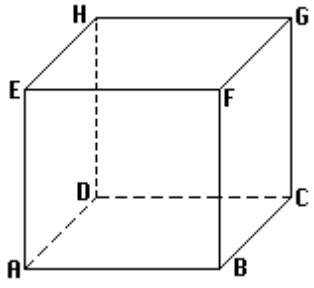
10. In the figure, $A\hat{B}C = 60°$, $AB = 4$ cm, $BC = 10$ cm, $CD = 6$ cm and $AD = 5.5$ cm. Find the value of $A\hat{D}C$.



(A) 60° (B) 98.5°
(C) 101.5° (D) 120°
(E) 125°

( )

11. The figure shows a cube. Which of the following is/are true?

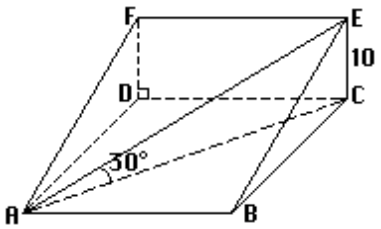


(I) $G\hat{A}C = 45°$ (II) $A\hat{C}H = 60°$
(III) $H\hat{A}B = 90°$

(A) I only (B) II only
(C) III only (D) I and III only
(E) II and III only

( )

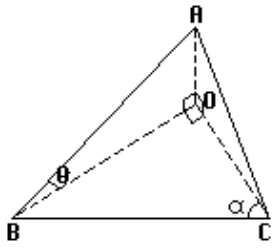
12. In the figure *ABCD*, *ABEF* and *CDFE* are rectangles. Given that *CDFE* is perpendicular to *ABCD*, *CE* = 10 cm, $E\hat{A}C = 30°$ and $E\hat{B}C = 45°$, calculate the length of *AB*.



(A) 10 cm (B) $10\sqrt{2}$ cm
(C) $10\sqrt{3}$ cm (D) 20 cm
(E) $20\sqrt{2}$ cm

( )

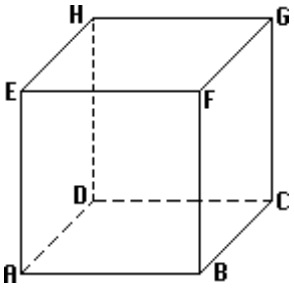
13. In the figure, *OBC* is a right-angled triangle in a horizontal plane with $B\hat{O}C = 90°$. *OA* is vertical, $A\hat{B}O = \theta°$, $B\hat{C}A = \alpha°$ and *OC* = 8 cm. The length of *AB* is given by



(A) $\frac{8 \sin \alpha}{\sin \theta}$ (B) $\frac{8 \sin \theta}{\sin \alpha}$ (C) $\frac{8 \cos \alpha}{\tan \alpha}$
(D) $\frac{8 \tan \alpha}{\cos \theta}$ (E) $\frac{8 \tan \theta}{\cos \alpha}$

( )

14. The figure shows a cube. Find $A\hat{F}H$.



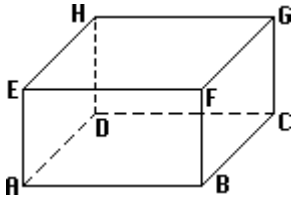
(A) 30° (B) 45°
(C) 60° (D) 75°
(E) 90°

( )

15. From the top of a building 100 m high, the angles of depression of two cars *P* and *Q* are 30° and 45° respectively. If *P* is due south and *Q* is due east of the building, find the distance in metres between *P* and *Q*.

(A) 200 m (B) 158 m (C) 120 m (D) 115 m (E) 72 m ( )

16. The figure shows a rectangular cuboid with *AB* = 8 cm, *BC* = 6 cm and *CG* = 5 cm. The value of $G\hat{A}C$ is



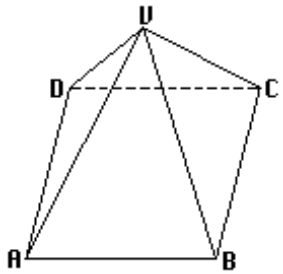
(A) 26.6° (B) 29.5°
(C) 30° (D) 33.3°
(E) 66.7°

( )

17. The angle of elevation of the top of a tower, from a point *A* on the horizontal ground north of the building is 45°. From another point *B*, 50 m east of *A*, the angle of elevation becomes 30°. Find the height of the building in metres.

(A) 25 (B) $25\sqrt{2}$ (C) $25\sqrt{3}$ (D) 50 (E) $50\sqrt{2}$ ( )

18. The figure shows a right pyramid with a square base *ABCD* of length 20 cm. If the length of a slant edge *VA* is 26 cm, find $V\hat{A}C$ correct to the nearest degree.



(A) 29° (B) 30°
(C) 33° (D) 57°
(E) 60°

( )

Answers

1. C	2. B	3. D	4. D	5. B
6. B	7. D	8. E	9. C	10. B
11. E	12. B	13. D	14. C	15. A
16. A	17. B	18. D		

XYZ SECONDARY SCHOOL

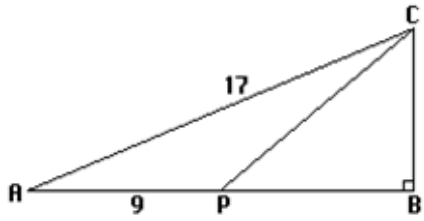
Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: ☐

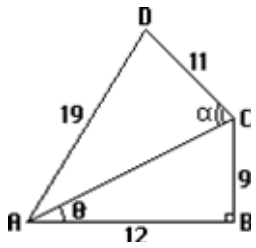
Secondary 3 Mathematics Test
Chapter 11 Further Trigonometry

1. In the diagram, *APB* is a straight line, $A\hat{B}C = 90°$, *AC* = 17 cm, *AP* = 9 cm and area of $\Delta APC = 36\ cm^2$.



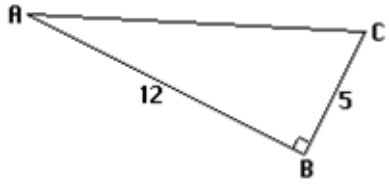
Calculate
(a) *BC*, [1]
(b) tan $C\hat{P}B$. [2]

2. In the diagram, *AB* = 12 cm, *BC* = 9 cm, $A\hat{B}C = 90°$, *CD* = 11 cm and *AD* = 19 cm.



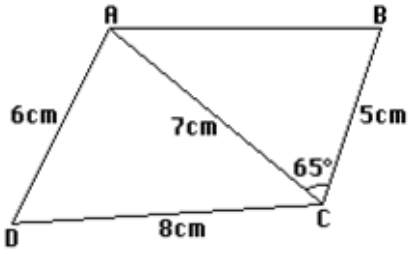
Calculate the value of
(a) sin θ
(b) cos α.
Give your answers as a fraction in its lowest terms. [3]

3. In the figure, *AB* = 12 cm, *BC* = 5 cm and $A\hat{B}C = 90°$.



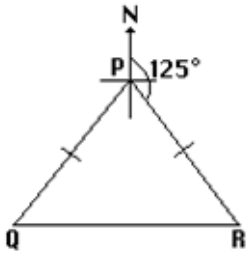
(a) Find the value of sin $B\hat{A}C$. [2]
(b) *A* circle is to be drawn so that it will pass through *A*, *B* and *C*. Write down the radius of the circle. [1]

4. In the diagram, *ABCD* is a quadrilateral in which *AD* = 6 cm, *DC* = 8 cm, *AC* = 7 cm, *BC* = 5 cm and $A\hat{C}B$ = 65°. Calculate



(a) $A\hat{C}D$. [2]
(b) the area of the triangle *BCD*. [2]

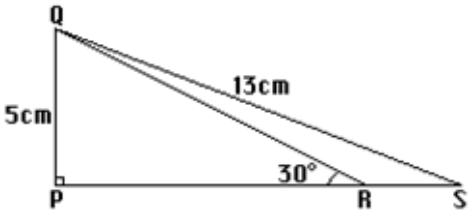
5. Three points *P*, *Q* and *R* are such that the bearing of *R* from *P* is 125°, the bearing of *Q* from *P* is 205°, and *PQ* = *PR*. Calculate



(a) the bearing of *P* from *R*, [1]
(b) the bearing of *P* from *Q*, [1]
(c) the length of *PQ* given that *QR* = 20 m. Give your answer correct to 1 decimal place. [2]

[Given sin 40° = 0.64, cos 40° = 0.77, tan 40° = 0.84]

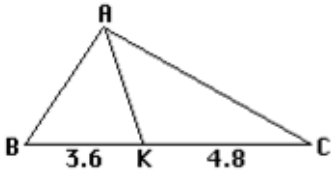
6. In the diagram, triangle *PQS* has a right angle at *P*, and *R* is a point on *PS* such that $P\hat{R}Q$ =30°. Given that *PQ* = 5 cm, and *QS* = 13 cm,



(a) find sin $P\hat{Q}S$, [2]
(b) find the length of *SR*, giving your answer in the form $a + b\sqrt{3}$ cm. [3]

7. In the triangle *ABC*, *AB* = 6.4 cm, *AC* = 8.3 cm, and $B\hat{A}C$ = 42.6°. Find the length of *BC* and the area of Δ*ABC*. [5]

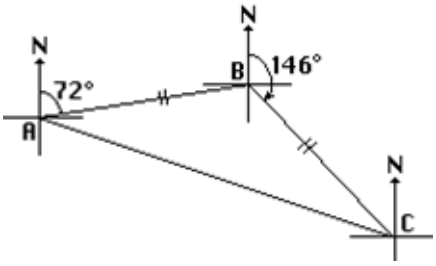
8. In Δ*ABC*, *K* is a point on *BC* such that *BK* = 3.6 cm and *CK* = 4.8 cm. Given that the area of Δ*ABK* = 9 cm^2 and sin $A\hat{B}C$ = 0.5, calculate



(a) the area of Δ*AKC*, [1]
(b) the length of *AB*. [2]

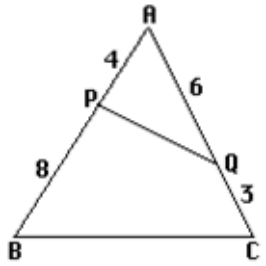
9. A ship, *P*, locates a lighthouse, *Q*, on a bearing of 300°, and another lighthouse, *R*, on a bearing of 030°. If *PQ* = 16 km and *PR* = 12 km, find the distance of *QR*. [3]

10. In the diagram, the bearing of *B* from *A* is 072°, and the bearing of *C* from *B* is 146°. If *AB* = *BC*, find the bearing of



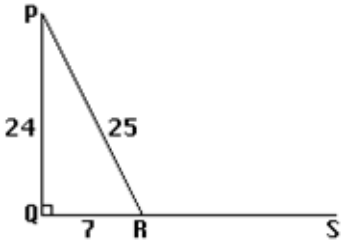
(a) *A* from *B*, [2]
(b) *A* from *C*. [2]

11. In the diagram, *AQ* = 6 cm, *QC* = 3 cm, *AP* = 4 cm, *PB* = 8 cm and the area of $\Delta APQ = 10\ cm^2$. Calculate



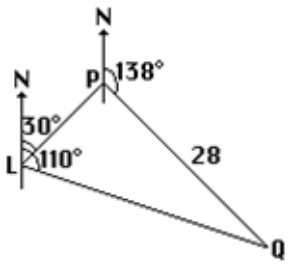
(a) sin $P\hat{A}Q$, [2]
(b) the area of the quadrilateral *BPQC*. [2]

12. In the diagram, *QRS* is a straight line. *PQ* = 24 cm, *QR* = 7 cm and *PR* = 25 cm.



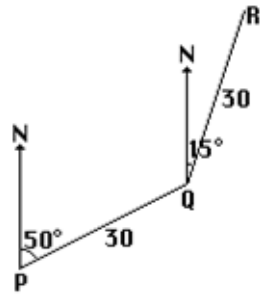
(a) Explain why $P\hat{Q}R = 90°$. [1]
(b) Express each of the following as a fraction and write down the value of
(i) sin $Q\hat{P}R$ [1]
(ii) tan $P\hat{R}Q$ [1]
(iii) cos $P\hat{R}S$ [1]

13. In the diagram, the bearings of two ships *P* and *Q* from a lighthouse *L* are 030° and 110° respectively. Given that *P* is 28 km from *Q* and that the bearing of *Q* from *P* is 138°, find



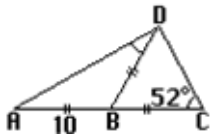
(a) the distance of *Q* from *L*, [3]
(b) the bearing of *L* from *Q*. [1]

14. A ship steams 30 km from a port P on a bearing of 050° to the point Q and then 30 km on a bearing of 015° from port Q to the port R.



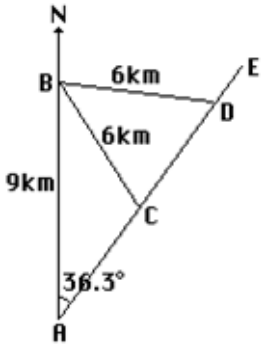
(a) Find the bearing of port R from port P, [2]
(b) Calculate the distance of PR. [2]
(c) Calculate the shortest distance from Q to the line joining P and R. [2]

15. In the diagram, $AB = BC = BD = 10$ cm and $B\hat{C}D = 52°$. Calculate



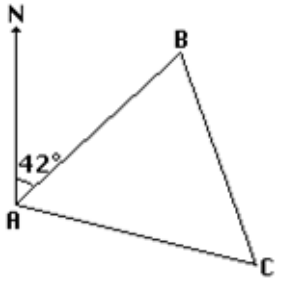
(a) $A\hat{D}B$, [2]
(b) the length of CD, [3]
(c) the length of AD. [3]

16. The figure shows a point A which lies 9 km south of point B. The points C and D are both 6 km from B and the bearing of C from A is 036.3°. The points A, C, D and E all lie on a straight line. Calculate



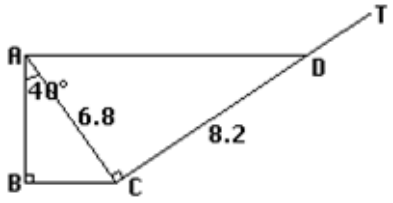
(a) $A\hat{C}B$, [3]
(b) the bearing of C from B, [1]
(c) the length of CD, [2]
(d) the shortest distance from B to the line ACD. [2]

17. In the figure, the points A, B and C form an equilateral triangle, and the bearing of B from A is 042°. Find



(a) the bearing of C from A, [1]
(b) the bearing of C from B. [2]

18. In the diagram, $A\hat{B}C = A\hat{C}D = 90°$, $B\hat{A}C = 40°$, $AC = 6.8$ cm and $CD = 8.2$ cm.



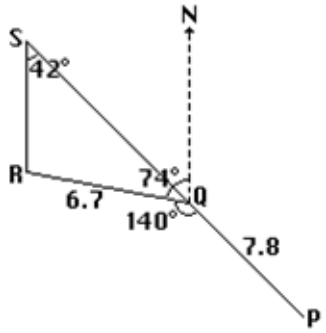
Calculate

(a) AB [2]

(b) cos $A\hat{D}C$ [2]

(c) sin $A\hat{D}T$ [2]

19. A ship sails 7.8 km from P to Q. It then sails another 6.7 km from Q to R. The ship finally sails from R to a point S, which is due north of Q. Given that $P\hat{Q}R = 140°$, $R\hat{Q}N = 74°$ and $R\hat{S}Q = 42°$, calculate



(a) the bearing of P from Q, [1]

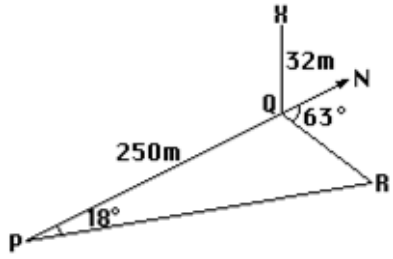
(b) the bearing of Q from R, [2]

(c) the distance of PR, [3]

(d) the distance of RS, [3]

(e) the shortest distance from Q to the ship as it sails from P to R. [3]

20. The points P, Q and R are on level ground such that Q is due north of P, the bearing of R from P is 018° and the bearing of R from Q is 063°.



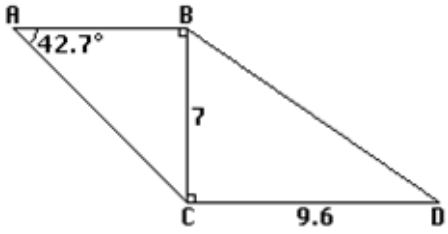
(a) Given that the distance $PQ = 250$ m, calculate

(i) the distance QR, [3]

(ii) the bearing of P from R. [3]

(b) Given that the vertical post XQ is 32 m high, calculate the angle of elevation of X from P. [2]

21. In the figure, $A\hat{B}C = B\hat{C}D = 90°$, $B\hat{A}C = 42.7°$, $BC = 7$ cm and $CD = 9.6$ cm. Calculate



(a) $B\hat{D}C$, [2]

(b) the length of AC, [2]

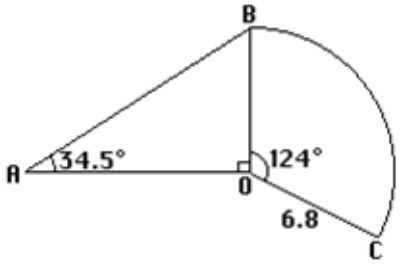
(c) the length of BD. [2]

Given that K is a point on BD such that CK is perpendicular to BD, calculate

(d) the length of CK. [3]

22. From a lighthouse *A*, the bearing of two ships *B* and *C* are 046° and 320° respectively. If *AB* = 458 m and *AC* = 625 m, calculate the distance of *BC* and the bearing of *B* from *C*. [5]

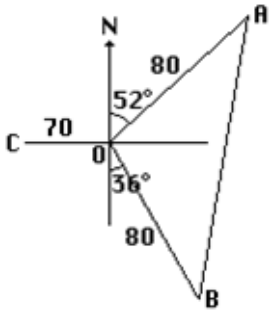
23. **(a)** In the figure, *O* is the centre of the sector *BOC*. Given that $B\hat{O}C$ = 124°, $B\hat{A}O$ = 34.5°, $A\hat{O}B$ = 90° and *OC* = 6.8 cm, calculate



(i) *AO*,
(ii) *AB*,
(iii) the area of sector *BOC*. [7]

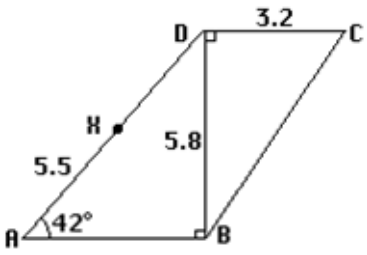
(b) Three points *A*, *B* and *C* lie on level ground. The bearing of *B* from *A* is 057° and the bearing of *C* from *A* is 126°. If *BC* = 84 m and *AB* = 65 m, find the bearing of *B* from *C*. [5]

24. In the diagram, *OA* = *OB* = 80 m, *OC* = 70 m, the bearing of *A* and *B* from *O* are 052° and 144° respectively.



(a) Calculate the bearing of *B* from *A*. [2]
(b) Calculate the length of *AC*. [3]
(c) A man walks from *A* to *C* until he reaches a point *P* where *OP* is a minimum. Calculate the length of *AP*. [3]

25. In the diagram, $A\hat{B}D = B\hat{D}C$ = 90°, $B\hat{A}D$ = 42°, *BD* = 5.8 m and *CD* = 3.2 m. Calculate

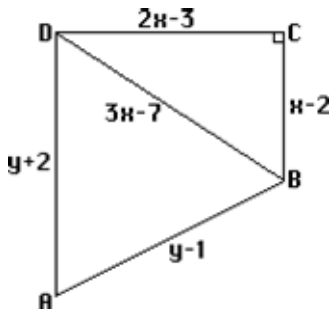


(a) *AD*,
(b) $C\hat{B}D$,
(c) *BC*.

Given that *X* is a point on *AD* such that *AX* = 5.5 m,
(d) calculate the area of Δ*ABX*. [6]

26. **(a)** Solve the equation $5x^2 + 2x = 9$, giving your answer correct to 3 significant figures. [3]

(b) In the diagram, $AB = (y - 1)$ cm, $BC = (x - 2)$ cm, $CD = (2x - 3)$ cm, $AD = (y + 2)$ cm, $BD = (3x - 7)$ cm and $B\hat{C}D = 90°$.

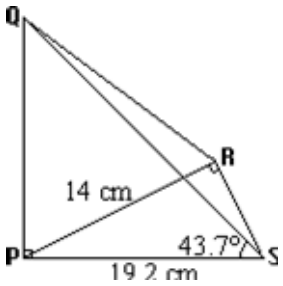


(i) Using the right-angled triangle BCD, form an equation in x, and hence find the value of x. [3]

(ii) If $BD = AD$, use your result in (i) to find y, and hence find $A\hat{D}B$. [3]

(iii) Find the area of ΔABD. [3]

27. In the diagram, $Q\hat{P}S = P\hat{R}S = 90°$, $Q\hat{S}P = 43.7°$, $PR = 14$ cm and $PS = 19.2$ cm. Calculate

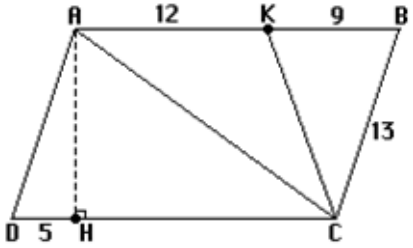


(a) $Q\hat{S}R$, [2]

(b) PQ, [2]

(c) QS. [2]

28. $ABCD$ is a parallelogram in which $BC = 13$ cm. H and K are points on DC and AB such that $AK = 12$ cm, $KB = 9$ cm, and $DH = 5$ cm.



Calculate

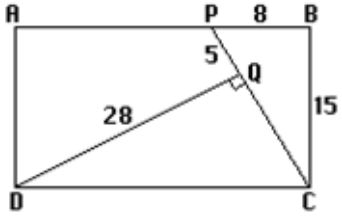
(a) AH, [1]

(b) $A\hat{B}C$, [2]

(c) the area of ΔKBL, where L is a point on BC such that $LC = 8$ cm. [3]

(d) the ratio of the area of ΔADH to the area of ΔACK. [1]

29. In the diagram, $ABCD$ is a rectangle, and DQ is perpendicular to CP. Given that $BC = 15$ cm, $PB = 8$ cm, $PQ = 5$ cm and $DQ = 28$ cm, calculate

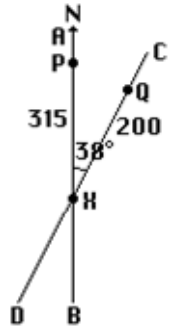


(a) CQ,

(b) AB,

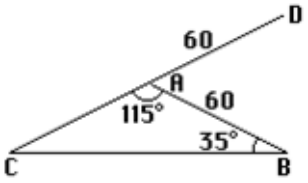
(c) $A\hat{D}Q$. [6]

30. *AB* and *CD* are two roads which meet at *X*. *AB* runs in a north-south direction and makes an angle of 38° with *CD*. At 09 45, a car *P*, which is heading south towards *B*, is 315 km north of the point *X*. At the same time, a truck *Q*, travelling along *CD*, is 200 km N 38°E of *X*. Given that the car *P* is travelling at 75 km/h, and the truck *Q* is travelling at 60 km/h, calculate



(a) the time at which the car passes *X*, [2]
(b) the distance between *P* and *Q* at 09 45, [3]
(c) the distance of *P* from *X*, and the distance of *Q* from *X*, at 15 57, [3]
(d) the bearing of *Q* from *P* at 15 57, giving your answer correct to the nearest degree. [4]

31. The figure shows the positions *A*, *B*, *C* and *D* of four oil rigs. *C*, *A* and *D* lie in a straight line. Given that *AD* = *AB* = 60 km, $C\hat{A}B$ = 115°, $A\hat{B}C$ = 35° and *B* is due east of *C*, calculate

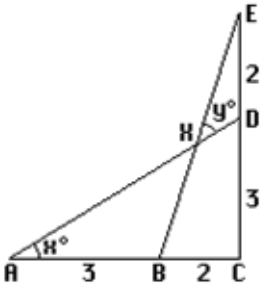


(a) the distance of *CB*, [3]
(b) the distance of *BD*. [3]

A supply ship *S* sets sail from *C* to *B* in a straight line.

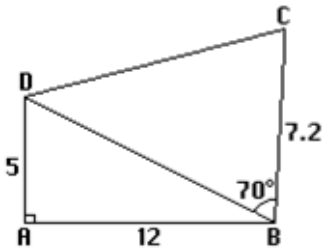
(c) Find the distance the ship *S* must move such that it will be closest to *A*. [3]

32. In the diagram, *AB* = *CD* = 3 cm and *BC* = *D*E = 2 cm. Calculate



(a) the length of *B*E, [2]
(b) the length of *AD*, [2]
(c) the values of *x* and *y*. [4]

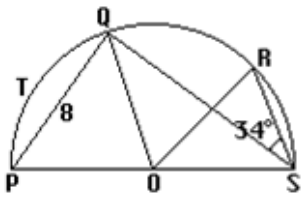
33. In the diagram, *AB* = 12 cm, *AD* = 5 cm, *BC* = 7.2 cm, $B\hat{A}D$ = 90° and $D\hat{B}C$ = 70°.



Calculate

(a) the length of *BD*, [2]
(b) the length of *CD*, [4]
(c) the area of Δ*DBC*. [3]

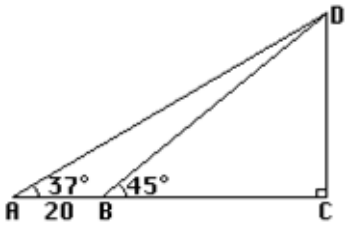
34. *P, T, Q, R* and *S* are points on the semicircle with centre *O* and *PS* diameter. The radius of the semicircle is 8.5 cm, the chord *PQ* is 8 cm and $Q\hat{S}R = 34°$.



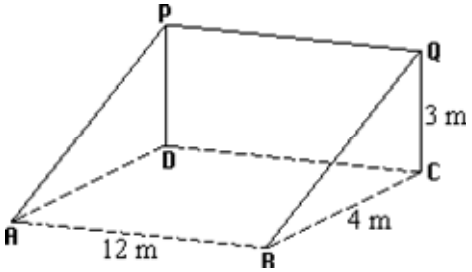
Calculate

(a) $Q\hat{O}R$, [1]
(b) the length of the chord *QS*, [2]
(c) the area of ΔPQS, [2]
(d) the perpendicular distance from *Q* to *PS*, [2]
(e) *QPO*, [2]
(f) the area of the minor segment *PTQ*. [3]

35. In the figure, $B\hat{C}D = 90°$, $B\hat{A}D = 37°$, $D\hat{B}C = 45°$ and *AB* = 20 m. Find the length of *CD*. [5]



36. The figure shows a section of a wall. The faces *ABCD* and *PQCD* are rectangular with *ABCD* on the horizontal and *PQCD* vertical. Given that *AB* = 12 m, *BC* = 4 m and *CQ* = 3 m,

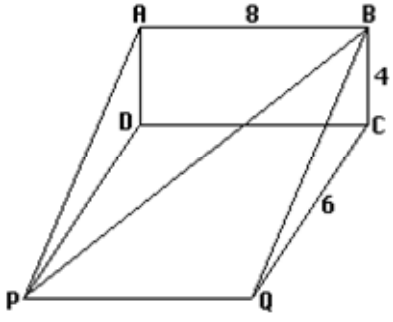


(a) find the length of *AQ*, [2]

(b) write down the numerical value of

(i) cos $C\hat{B}Q$, [1]
(ii) sin $C\hat{A}Q$. [1]

37. The diagram shows a rectangular vertical billboard being supported by straight wires *AP*, *BQ* and *PB*. Given that *AB* = 8 m, *BC* = 4 m and *CQ* = 6 m,

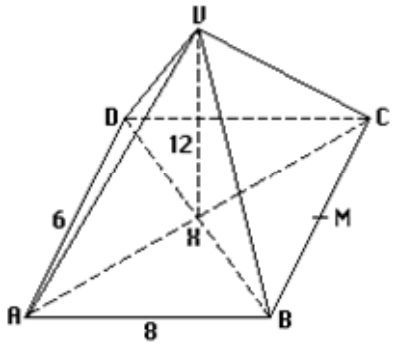


(a) calculate the length of *PC*, [1]

(b) write down the numerical value of

(i) tan $B\hat{P}C$, [1]
(ii) sin $D\hat{P}C$. [1]

38. The figure shows a pyramid with a horizontal rectangular base $ABCD$ where AB = 8 cm, AD = 6 cm and the vertex V is 12 cm vertically above X. The mid-point of BC is M.



Calculate

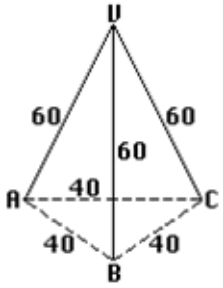
(a) the length of VM, [2]

(b) tan $V\hat{A}X$, [1]

(c) sin $A\hat{V}X$. [2]

[Given that $\sqrt{10}$ = 3.16, $\sqrt{153}$ = 12.37, $\sqrt{12}$ = 3.46]

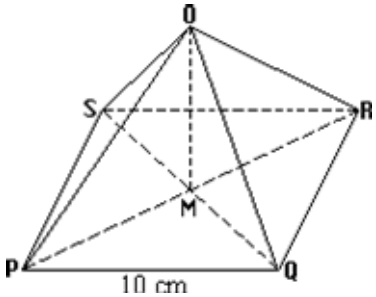
39. The legs of a camera tripod are each 60 cm long. When it stands on horizontal ground, the ends of the legs form a equilateral triangle of side 40 cm. Find



(a) the height of the vertex V above the ground, [3]

(b) the angle made by each leg with the ground. [2]

40. The figure shows a pyramid with a square base PQRS and vertex O. The diagonals of the base intersect at M. OP = OQ = OR = OS = 16 cm and PQ = 10 cm.



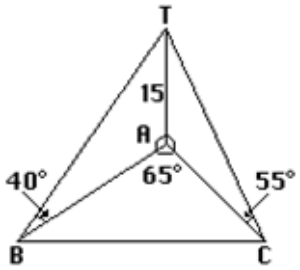
Calculate

(a) PR, [2]

(b) OM, [2]

(c) $P\hat{O}$R. [2]

41. In the diagram, A, B, and C are three points on level ground. AT is a vertical pole of height 15 m, the angle of elevation of T from B is 40° and the angle of elevation of T from C is 55°. If $B\hat{A}C$ = 65°, calculate



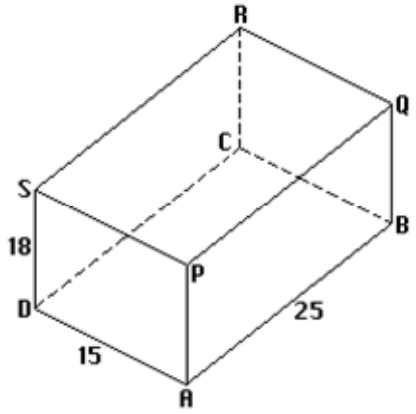
(a) the length of AB, [2]

(b) the length of AC, [2]

(c) the area of ΔABC, [2]

(d) the length of BC. [3]

42. The figure shows a rectangular box of dimension 25 cm × 15 cm × 18 cm.

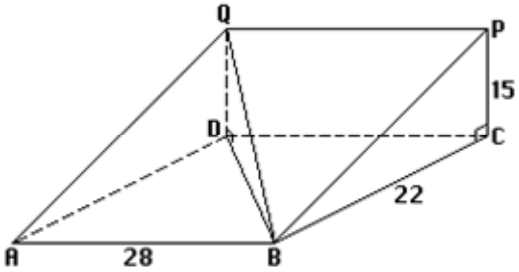


(a) Calculate the length of *A*R, giving your answer correct to 1 decimal place. [3]

(b) Find the angle that *AQ* makes with the horizontal i.e. *QÂB*. [2]

(c) Find $P\hat{C}A$. [3]

43. The diagram represents a solid in which the horizontal base *ABCD* and the vertical face *PQDC* are rectangles. Given that *AB* = 28 cm, *BC* = 22 cm and *PC* = 15 cm, calculate

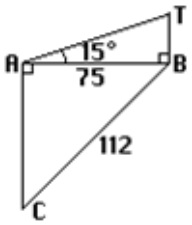


(a) the length of *BD* and *BQ*, giving your answer correct to 1 decimal place, [3]

(b) the angle that the line *BP* makes with the horizontal, i.e. $P\hat{B}C$, [2]

(c) the angle that the line of greatest slope *BQ* makes with the horizontal, i.e. $D\hat{B}Q$. [2]

44. In the diagram, *B* is the foot of a vertical pole *BT*. *A*, *B* and *C* are on horizontal ground where *AB* = 75 m, *BC* = 112 m and *BÂC* = 90°. Given that the angle of elevation of *T* from *A* is 15°, calculate



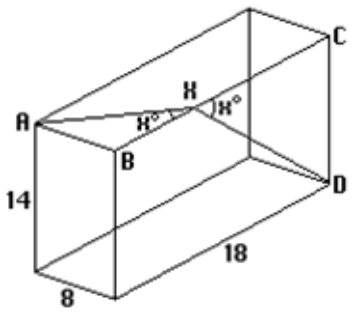
(a) the height of the pole, [2]

(b) *ABC*, [2]

(c) the shortest distance from *A* to *BC*, [2]

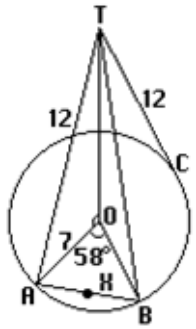
(d) the angle of elevation of *T* from *C*. [2]

45. In the diagram, *A*, *B*, *C* and *D* are four corners of a rectangular block of dimension 18 cm by 8 cm by 14 cm and *X* is a point on *BC* such that $A\hat{X}B = C\hat{X}D = x°$. Calculate



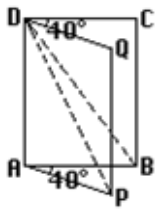
(a) the length of *AD*, [3]
(b) the value of *x*, [3]
(c) the length of *BX*. [2]

46. The figure shows a horizontal ring with centre *O* and radius *T* cm being suspended from a point *T* by three strings *TA*, *TB* and *TC*, each of length 12 cm. Given that $A\hat{O}B = 58°$ and *X* is the mid-point of *AB*, find



(a) the length of *TO*, [2]
(b) the angle of elevation of *T* from *A*, [2]
(c) the length of *OX*, [2]
(d) the angle of elevation of *T* from *X*, [2]
(e) $A\hat{T}B$. [4]

47. The diagram shows a door of dimensions 2.2 m by 1.2 m turning through 40° from position *ABCD* to *APQD*. Calculate

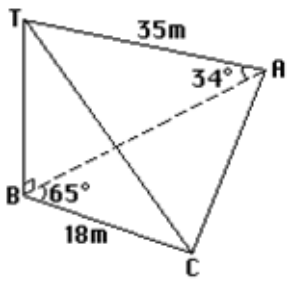


(a) the length of *PB*, [3]

(b) the angle of elevation of *C* from *P* [3]

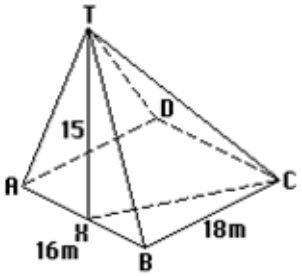
(c) $P\hat{D}B$. [3]

48. The diagram shows a triangle *ABC* lying in a horizontal plane. *T* is a pole vertically above *B*. Given that *BC* = 18 m, *AT* = 35 m, $B\hat{A}T = 34°$ and $A\hat{B}C = 65°$, calculate



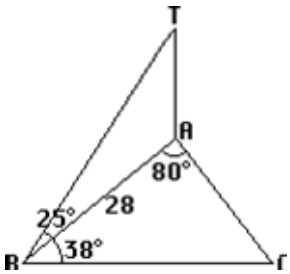
(a) the length of *AC*, [4]
(b) the length of *TC*, [3]
(c) the angle of elevation of *T* from *C*. [2]

49. In the diagram, *A*, *B*, *C* and *D* are four points on the corner of a rectangle. Given that *AB* = 16 m, *BC* = 18 m, *X* is the mid-point of *AB* and *T* is a point vertically above *X*. If *TX* = 15 m, calculate



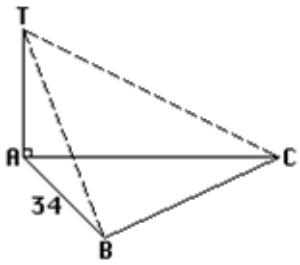
(a) *CX*, [2]
(b) the angle of elevation of *T* from *D*, [2]
(c) the angle of elevation of *T* from *B*, [2]
(d) the length of *CT*. [2]

50. The figure shows three points *A*, *B*, and *C* with a vertical mast *AT* standing at *A*. The angle of elevation of *T* from *B* is 25°. Given that *AB* = 28 m, $A\hat{B}C$ = 38° and $B\hat{A}C$ = 80°, calculate



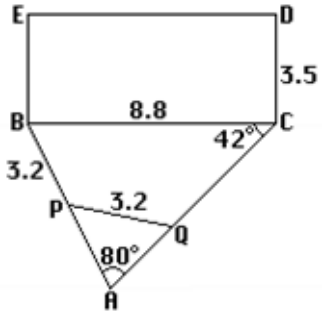
(a) the height of the mast correct to 2 decimal places, [2]
(b) the length of *AC*, [3]
(c) the angle of elevation of *T* from *C*, [3]
(d) the length of *BC*. [3]

51. In the diagram, *A*, *B* and *C* are three points on the horizontal plane. *AT* is a vertical tower. The angles of depression of *B* and *C* from the top of the tower are 35° and 18° respectively. Given that *AB* = 34 m and $B\hat{A}C$ = 56°, calculate



(a) the height of the tower, [2]
(b) the length *AC*, [2]
(c) the area of ΔABC, [2]
(d) the length of *BC*, [3]
(e) the greatest angle of elevation of *T* from a point on *BC*. [3]

52. In the diagram, *BCDE* represents a vertical rectangular billboard of height 3.5 m. *A*, *B* and *C* are three points on level ground such that *BC* = 8.8 m, $B\hat{A}C = 80°$ and $A\hat{C}B = 42°$. *P* is a point on *AB* and *Q* is a point on *AC* such that *BP* = *PQ* = 3.2 m.

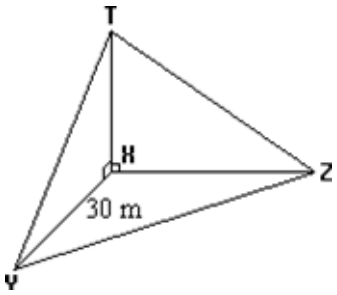


(a) Calculate the lengths of *AP* and *CP*. [5]

(b) Calculate the size of $P\hat{Q}C$. [3]

(c) A cat walks along the top edge of the billboard from *E* to *D*. Find the largest possible angle of elevation of the cat from *P*. [3]

53. Three points *X*, *Y* and *Z* are on level ground. *Y* is due south of *X* and *Z* is due east of *X*. *XT* is a vertical flag pole and *XY* = 30 m.



(a) Given that the angle of elevation of *T* from *Y* is 28°, calculate the height of the flag pole, giving your answer correct to one decimal place. [2]

(b) The bearing of *Z* from *Y* is 042°. Calculate the distance of *XZ* and *YZ*. [4]

(c) Calculate the angle of elevation of *T* from *Z*. [3]

(d) A man walks from *Y* to *Z*. Calculate the distance he must walk to a point *K* on *YZ* so that the angle of elevation of *T* from *K* will be the greatest. [3]

54. Given that $\sin\theta = \frac{1}{3}$ and $90° < \theta < 180°$, find the value of

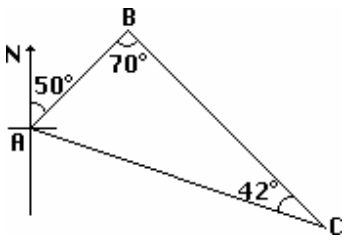
(a) $\cos\theta$ **(b)** $\tan\theta$ [3]

55. Given that $\cos x° = \sin y° = \sin 25°$ and that *x* is acute and *y* is obtuse, find the value of *x* and of *y*. [3]

56. (a) If $\sin x° = \sin 23°$ and $90 < x < 180$, find *x*.

(b) If $\cos x° = -\sin 60°$ and $0 < x < 180$, find *x*. [3]

57. The diagram shows the positions of *A*, *B* and *C*. Find the bearing of



(a) *B* from *A*, **(b)** *C* from *A*,

(c) *C* from *B*, **(d)** *A* from *C*. [4]

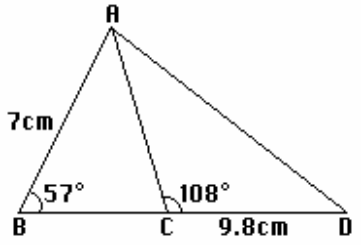
58. In ΔABC, AB = 7 cm, BC = 6 cm and AC = 8 cm. Find the value of cos $B\hat{A}C$, leaving your answer as a fraction in its lowest term. [3]

59. (a) In Δ ABC, $B\hat{A}C$ = 26°, $A\hat{B}C$ = 61° and AC = 10 cm. Calculate the perimeter of ΔABC, giving your answer correct to 3 significant figures. [3]

(b) In Δ PQR, PQ = 7.5 cm, PR = 8.8 cm and $Q\hat{P}R$ = 72°. Calculate the length of QR and the value of cos $P\hat{Q}R$. [3]

60. In the diagram, AB = 7 cm, CD = 9.8 cm, $A\hat{B}C$ = 57° and $A\hat{C}D$ = 108°. Calculate

(a) BC, **(b)** AC,
(c) the area of Δ ABD. [7]



61. In ΔABC, AB = 7.2 cm, AC = 8.8 cm and $B\hat{A}C$ = 132°. Calculate

(a) the length of BC, **(b)** the area of ΔABC. [5]

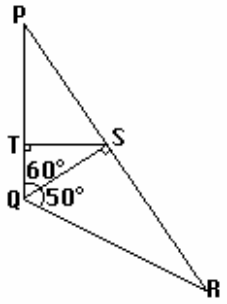
62. Two ships leave a port at the same time. One sails at 22 km/h on a bearing of 047° and the other at 18 km/h on a bearing of 148°. Find the distance between the ships after 3 hours. [4]

63. The bearing of a point B from A is 023.2° and the bearing of C from A is 264°. Given that AB = 225 m and AC = 186 m, calculate

(a) BC, **(b)** the bearing of C from B. [5]

64. In the diagram, QS and ST are perpendicular to PR and PQ respectively. Given that $R\hat{Q}S$ = 50°, $P\hat{Q}S$ = 60° and PQ = 8 cm, calculate the length of

(a) RS, **(b)** PT. [5]



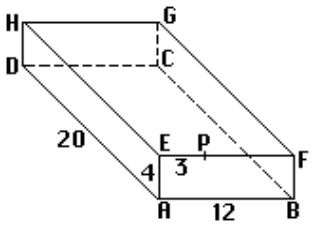
65. The diagram shows a rectangular cuboid where AB = 12 cm and AD = 20 cm, AE = 4 cm and P is a point on EF such that EP = 3 cm. Find

(a) the length of AP,

(b) tangent of $P\hat{A}E$,

(c) $P\hat{B}F$,

(d) $A\hat{B}D$. [8]



66. The diagram shows a pyramid with a rectangular base $ABCD$. It is given that AB = 16 cm, BC = 12 cm and V is vertically above the centre of the base N. If VN = 20 cm, calculate

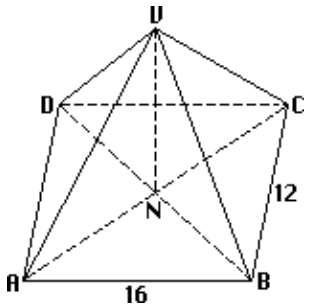
(a) the length of VA,

(b) tan $V\hat{A}N$,

(c) sin$V\hat{B}N$,

(d) $A\hat{V}C$,

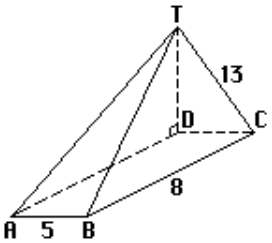
(e) $B\hat{V}D$. [10]



67. The diagram shows a rectangular pyramid $TABCD$ where TDC is vertical. Given that AB = 5 cm, BC = 8 cm and TC = 13 cm, calculate

(a) $T\hat{A}D$,

(b) $T\hat{B}D$. [6]



68. P, Q and R are three points on level ground. V is a point vertically above P. PQ = 9 m, PV = 12 m and VR = 24 m.

(a) Write down the sine of the angle between PR and VR.

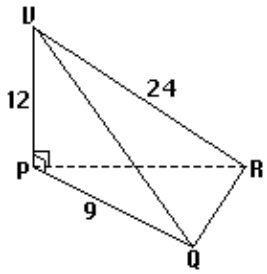
(b) Calculate PR.

(c) Write down the tangent of the angle between PQ and VQ.

(d) Given that $P\hat{Q}R$ = 120°, calculate

(i) $P\hat{R}Q$

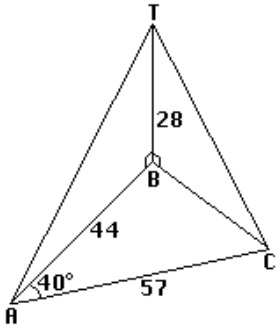
(ii) the length QR. [10]



69. The diagram shows 3 points *A*, *B* and *C* on level ground. *T* is a point vertically above *B*. Given that *AB* = 44 m, *AC* = 57 m, *TB* = 28 m and $B\hat{A}C = 40°$, calculate

(a) $T\hat{A}B$, **(b)** *BC*,

(c) $T\hat{C}B$. [7]

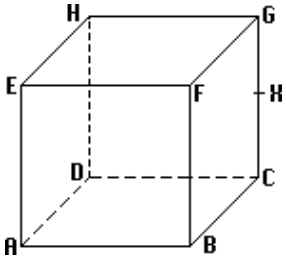


70. The figure shows a cube with edges 4 cm long. *ABCD* is horizontal and *X* is the mid-point of the vertical edge *CG*. Calculate

(a) sin $X\hat{A}C$, giving your answer as a fraction in its lowest term,

(b) the angle of elevation of *X* from *B*,

(c) $A\hat{X}E$. [6]

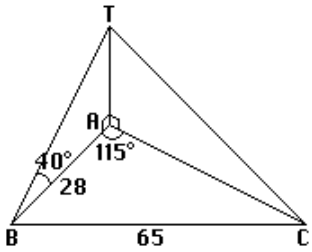


71. The figure shows a horizontal plot of ground *ABC* with a vertical pole *T* standing at the corner *A*. Given that $B\hat{A}C = 115°$, *AB* = 28 m, *BC* = 65 m and the angle of elevation of *T* from *B* is 40°, calculate

(a) the height of the pole,

(b) the length of *AC*,

(c) the length of *TC*. [8]

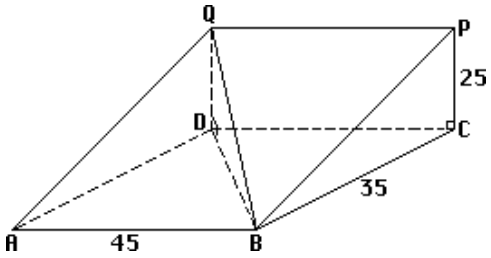


72. The figure represents a solid with a horizontal rectangular base *ABCD* and the vertical rectangular face *PQDC*. Given that *AB* = 45 cm, *BC* = 35 cm and *PC* = 25 cm, calculate

(a) the angle of elevation of *Q* from *A*,

(b) the angle of elevation *Q* from *B*,

(c) the length of *QB*. [8]



Answers

1. (a) 8 (b) $1\frac{1}{3}$

2. (a) $\frac{3}{5}$ (b) $-\frac{1}{22}$

3. (a) $\frac{5}{13}$ (b) 6.5 cm

4. (a) 46.6° (b) 18.6 cm^2

5. (a) 305° (b) 025° (c) 15.6 m

6. (a) $\frac{12}{13}$ (b) $12 - 5\sqrt{3}$ cm

7. 5.63 cm ; 17.98 cm^2

8. (a) 12 cm^2 (b) 10 cm

9. 20 km

10. (a) 252° (b) 289°

11. (a) $\frac{5}{6}$ (b) 35 cm^2

12. (b) (i) $\frac{7}{25}$ (ii) $\frac{24}{7}$ (iii) $-\frac{7}{25}$

13. (a) 27.04 km (b) 290°

14. (a) 032.5° (b) 57.2 km (c) 9.02 km

15. (a) 38° (b) 12.3 km (c) 15.8 km

16. (a) 117.4° (b) 153.7° (c) 5.52 km (d) 5.33 km

17. (a) 102° km (b) 162°

18. (a) 5.21 (b) 0.770 (c) 0.638

19. (a) 146° (b) 106° (c) 13.6 km (d) 5.31 km (e) 2.46 km

20. (a) (i) 109.25 m (ii) 198° (b) 7.3°

21. (a) 36.1° (b) 10.3 cm (c) 11.9 cm (d) 5.66 cm

22. 748.6 m ; 102.4°

23. (a) (i) 9.89 cm (ii) 12.0 cm (iii) 50 cm^2 (b) 352.3°

24. (a) 188° (b) 141.9 m (c) 76.22 m

25. (a) 8.67 m (b) 28.9° (c) 6.62 m , 11.85 m^2

26. (b) (i) 4.5 (ii) 4.5 , 31.2° (iii) 10.96 cm^2

27. (a) 3.1° (b) 18.3 cm (c) 26.6 cm

28. (a) 12 (b) 67.4° (c) 20.8 cm^2 (d) $\frac{5}{12}$

29. (a) 12 cm (b) 30.46 cm (c) 66.8°

30. (a) 13 57 (b) 199.8 km (c) *P*, 150km *Q*, 172 km (d) 278°

31. (a) 108.8 km (b) 64.5 km ; 59.6 km

32. (a) 5.39 cm (b) 5.83 cm (c) $x = 40.0$, $y = 37.2$

33. (a) 13 cm (b) 12.5 cm (c) 44 cm^2

34. (a) 68° (b) 15 cm (c) 60 cm^2
(d) $7\frac{1}{17}$ cm (e) 64.8° (f) 3.94 cm^2

35. 61.15 m

36. (a) 13 m (b) (i) $\frac{4}{5}$ (ii) $\frac{3}{13}$

37. (a) 10 m (b) (i) $\frac{2}{5}$ (ii) $\frac{4}{5}$

38. (a) 12.64 cm (b) $2\frac{2}{5}$ (c) $\frac{5}{13}$

39. (a) 55.4 cm (b) 67.4°

40. (a) 14.14 cm (b) 14.35 cm (c) 52.5°

41. (a) 17.88 m (b) 10.50 m (c) 85.1 m^2 (d) 16.47 m

42. (a) 34.3 cm (b) 35.8° (c) 31.7°

43. (a) 35.61 cm, 38.64 cm (b) 34.3° (c) 27.8°

44. (a) 20.1 m (b) 48.0° (c) 55.7 m (d) 10.2°

45. (a) 24.2 cm (b) 50.7° (c) $6\frac{6}{11}$ cm

46. (a) 9.75 m (b) 54.3° (c) 6.12 m
(d) 57.9° (e) 32.9°

47. (a) 0.82 m (b) 69.5° (c) 18.9°

48. (a) 26.9 m (b) 26.6 m (c) 34.1°

49. (a) 19.7 m (b) 37.3° (c) 61.9° (d) 24.8 m

50. (a) 13.06 m (b) 19.52 m (c) 33.8° (d) 31.23 m

51. (a) 23.8 m (b) 73.3 m (c) 723.1 m^2
(d) 63.12 m (e) 46.1°

52. (a) 2.78 m, 7.60 m (b) 121.2° (c) 52.2°

53. (a) 16.0 m (b) 27.0 m, 40.4 m (c) 30.6° (d) 22.3 m

54. (a) $\frac{-2\sqrt{2}}{3}$ (b) $-\frac{\sqrt{2}}{4}$

55. $x = 65$, $y = 155$

56. (a) 157 (b) 150

57. (a) 050° (b) 118° (c) 160° (d) 298°

58. $\frac{11}{16}$

59. (a) 26.4 cm (b) 9.84 cm, 0.496

60. (a) 5.72 cm (b) 6.17 cm (c) 45.6 cm^2

61. (a) 14.6 cm (b) 23.5 cm^2

62. 92.9 km

63. (a) 355 m (b) 230.4°

64. (a) 4.77 cm (b) 6 cm

65. (a) 20.2 cm (b) $\frac{3}{20}$ (c) 66.0° (d) 59.0°

66. (a) 24.5 (b) 2 (c) 0.894 (d) 53.1° (e) 53.1°

67. (a) 56.3° (b) 51.8°

68. (a) $\frac{1}{2}$ (b) 20.8 m (c) $\frac{4}{3}$ (d) (i) 22° (ii) 14.8 m

69. (a) 32.5° (b) 36.6 m (c) 37.4°

70. (a) $\frac{1}{3}$ (b) 26.6° (c) 38.9°

71. (a) 23.5 m (b) 48 m (c) 41.9 m

72. (a) 35.5° (b) 23.7° (c) 62.2 m

Chapter 12

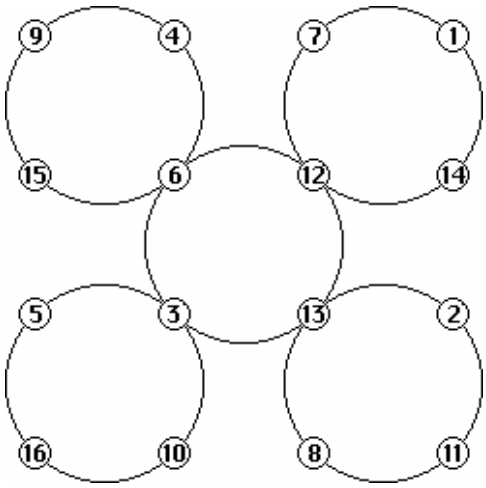
Secondary 3 Mathematics
Chapter 12 Mensuration – Arc Length, Sector Area, Radian Measure

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 341)
4 colours

Just For Fun (pg 343)
(a), (c)

Just For Fun (pg 352)



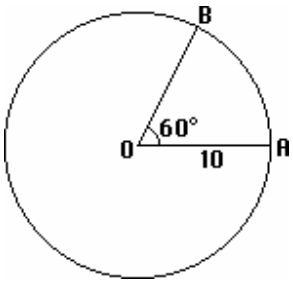
Secondary 3 Mathematics
Chapter 12 Mensuration – Arc Length, Sector Area, Radian Measure

GENERAL NOTES

The Exploration on page 340 will lead pupils to derive the formula for the arc length and area of a sector. This activity is useful for pupils to discover the formula.

Derive the relationship between radian measure and degree measure using the direct variation approach. This is also an easier way to derive the formulas for arc length and area of sectors, $S = r\theta$ and $A = \frac{1}{2}r^2\theta$.

As the pupils had learned length of arc formula and area of sector formula, it is good to give an example to show that what they are learning now is exactly the same as what they had learnt in Secondary Two.

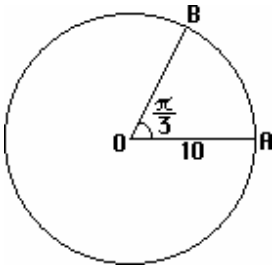


The diagram shows a circle with centre O, radius 10 cm with $A\hat{O}B = 60^0$. The arc length AB is given by

$$\frac{60^0}{360^0} \times 2\pi(10) \text{ cm} = 3\tfrac{1}{3}\pi \text{ cm}.$$

The area of the sector $AOB = \frac{60^0}{360^0} \times \pi(10^2) = 16\tfrac{2}{3}\pi \text{ cm}^2$.

Now the angle 60^0 is equal to $\frac{\pi}{3}$ radian.



$\therefore$ the arc length $AB = r\theta$

$= 10 \times \frac{\pi}{3}$ cm

$= 3\frac{1}{3}\pi$ cm

and the area of the sector $AOB = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(10^2)\frac{\pi}{3}$

$= 16\frac{2}{3}\pi \text{ cm}^2$.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

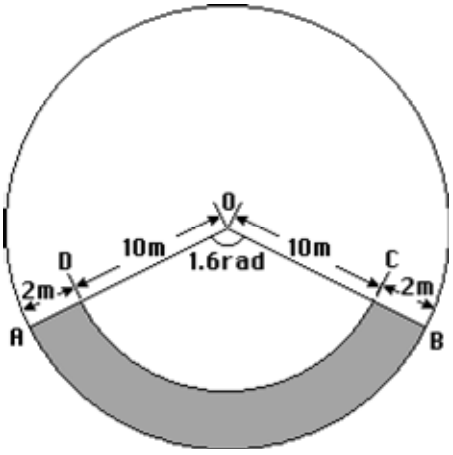
Time allowed: min

Class: _____ Marks: ☐

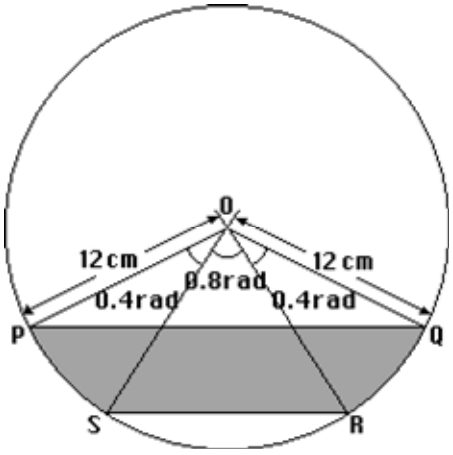
Secondary 3 Mathematics Test
Chapter 12 Mensuration – Arc Length, Sector Area, Radian Measure

1. In each of the following diagrams, find
   (a) the perimeter of the shaded region,
   (b) the area of the shaded region.

(i)

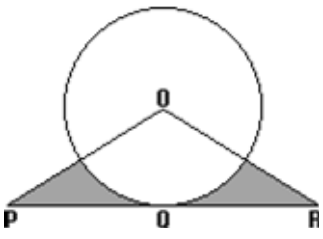


(ii)

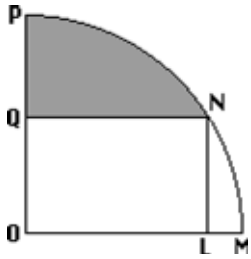


[8]

2. In the diagram, *PQR* is a tangent to the circle centre *O*, radius 10.2 cm, touching the circle at *Q*. Given that *OP* = *OR* and angle *POQ* = 1.2 radians, find
   (a) the perimeter of the shaded region,
   (b) the area of the shaded region. [6]

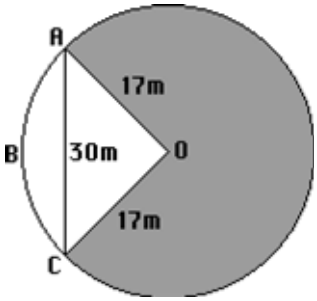


3. The diagram shows a rectangle *OLNQ* of side 5 cm and 12 cm in a quarter of a circle with centre *O*.
   (a) Show that angle *NOQ* is 1.176 radians to three decimal places.
   (b) Find the radius of the circle.
   (c) Calculate the area of the shaded region.

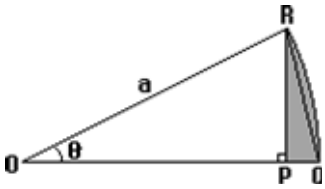


[3]

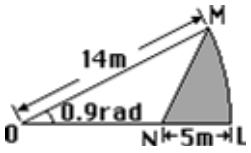
4. The diagram shows a circle of radius 17 m and a chord *AC* of length 30 m.
   Calculate
   (a) the length of arc *ABC*,
   (b) the area of the shaded region. [5]



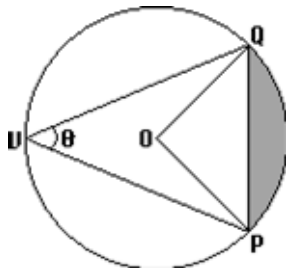
5. Given that *OQR* is a sector of a circle with angle *QOR* = θ radians, and *OR* = *a* units, show that the area of the shaded region is given by $\frac{1}{2}a^2(\theta - \sin\theta\cos\theta)$
   Find the area of the shaded region when *a* = 8 and θ = 0.6. [6]



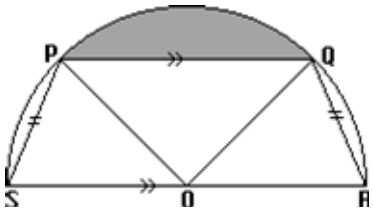
6. *OLM* is a sector of a circle, centre *O* and radius 14 m. *N* lies on *OL* such that *LN* = 5m. Given that angle *LOM* = 0.9 radian, calculate
   (a) the length of arc *LM*,
   (b) the length of *MN*,
   (c) the perimeter of the shaded region,
   (d) the area of the shaded region. [8]



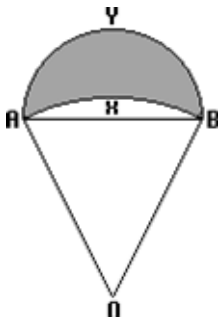
7. In the diagram, the radii *OP* and *OQ* are 8.4 cm and angle *PVQ* is θ radians. Given that the value of θ is 0.6, find
   (a) the length of arc *PQ*,
   (b) the area of the shaded region. [6]



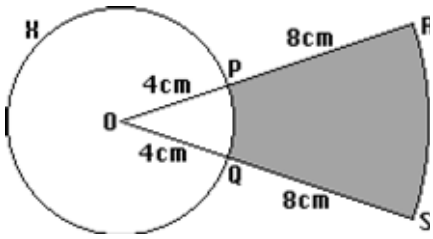
8. In the diagram, *O* is the centre of the semi-circle *PQRS*. The quadrilateral *PQRS* is a trapezium with *PQ* // *SR*. Given that the diameter *SR* = 14 cm and *PS* = *QR* = 5 cm, calculate
   (a) angle *POQ* in radians,
   (b) the length of arc *PS*,
   (c) the area of the shaded segment. [7]



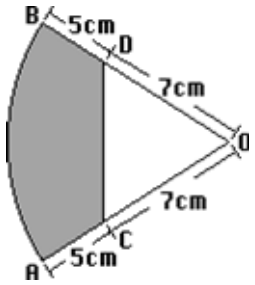
9. In the diagram, *AXB* is an arc of a circle centre *O* and radius 10 cm. *AYB* is a semicircle with *AB* as diameter. If triangle *AOB* is equilateral, calculate
   (a) the length of arc *AXB*,
   (b) the area of the minor segment *AXB*,
   (c) the area of the shaded region. [7]



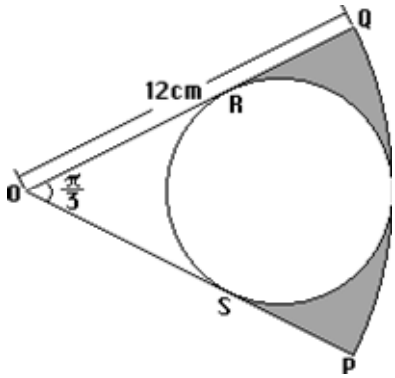
10. The diagram shows a circle *PXQ*, centre *O*, radius 4 cm. The radii *OP* and *OQ* are produced to *R* and *S* respectively so that *PR* = *QS* = 8 cm. An arc of a circle, centre *O*, radius 12 cm is drawn from *R* to *S*. If the area of the sector *ORS* is $\frac{4}{5}$ of the area of the circle *PXQ*, calculate
    (a) angle *POQ* in radians,
    (b) the perimeter of the shaded region,
    (c) the area of the shaded region. [7]



11. In the diagram, *OAB* is a sector of a circle centre *O* and radius 12 cm. The points *C* and *D* are on *OA* and *OB* respectively such that *OC* = *OD* = 7 cm. *CD* is a straight line. Given that the length of arc *AB* is 15 cm, calculate
(a) angle *AOB* in radians,
(b) the area of the shaded region. [5]



12. In the diagram, the radius of the sector *OPQ* is 12 cm and angle $POQ = \frac{\pi}{3}$.
Calculate
(a) the radius of the inscribed circle,
(b) the area of the shaded region. [6]



13. A wheel of radius 14 cm is turning at a rate of 6 revolutions per minute. Calculate, taking $\pi = \frac{22}{7}$,
(a) the angle through which the wheel turns in 1 second, [2]
(b) the distance moved by a point on the rim in 5 seconds. [2]

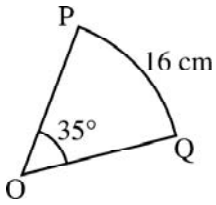
14. The lengths of the minor and major arcs of a circle are $51\frac{1}{3}$ cm and $80\frac{2}{3}$ cm respectively.
Taking $\pi = \frac{22}{7}$, find (a) the radius of the circle, [2]
(b) the angle subtended at the centre by the major arc. [2]

15. The arc length of a circle is 16 cm. The corresponding sector has an area of 96 cm^2. Find
(a) the radius of the circle, [3]
(b) the angle subtended at the centre of the circle by the arc. [2]

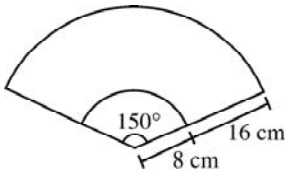
16. The length of the minute hand of a clock is 20cm. Find the distance which the tip of the minute hand moves in 38 minutes.
(Take $\pi = 3.14$ and correct your answer to 3 significant figures.) [3]

17. The minute hand of a clock is 10cm long. Find the area swept by the minute hand in 40 minutes. [3]

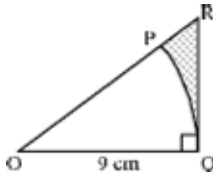
18. In the diagram, O is the centre of a circle with arc PQ = 16cm and $\angle POQ = 35^\circ$. Find the radius of the circle correct to 3 significant figures. [4]



19. A paper fan has the shape of a sector as shown in the diagram. Find the area of the shaded, region to the nearest cm^2. [3]



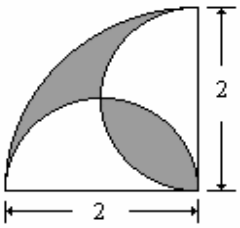
20. The diagram shows an arc PQ of a circle, centre O and radius 9 cm. Given that the perimeter of sector OPQ is 24.3cm and QR is a perpendicular to OQ, find the area of the shaded region. [4]



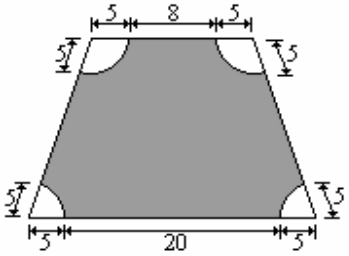
21. Find the area of a circular path 2 m wide surrounding a circular pond of radius 10 m. [3]

22. Find the areas of the shaded regions in the following diagrams. (All dimensions are in cm and all curves are circular.) [12]

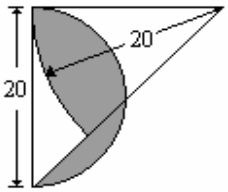
(a)



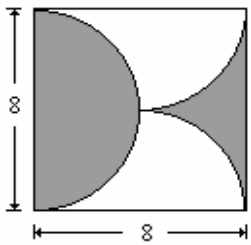
(b)



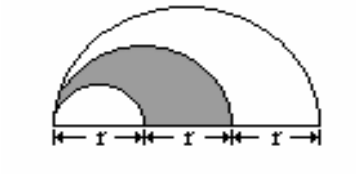
(c)



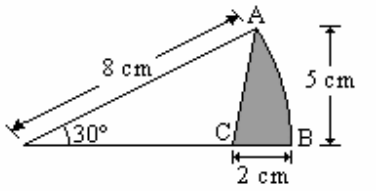
(d)



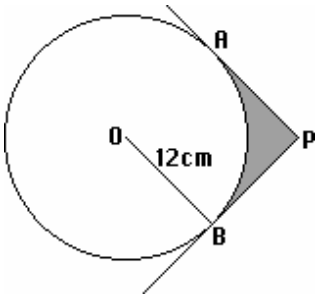
23. The diagram shows three semi-circles with radii *r*, *2r* and *3r* respectively. Find the ratio of the area of the shaded part to that of the unshaded part. [3]



24. In the diagram, *OAB* is a sector of a circle of radius 8 cm. $\text{A}\hat{\text{O}}\text{B} = 30°$ and *C* is a point on *OB* such that *CB* = 2 cm. Find the area of the shaded region given that *A* is 5 cm vertically above *OB*. [4]



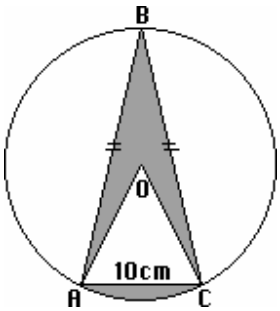
25. In the diagram, *PA* and *PB* are tangents to the circle centre *O* and radius 12 cm. If $A\hat{P}B = 1.38$ radians, calculate the area of the shaded region. [4]



26. The diagram shows a circle of radius 13 cm and centre *O*. *ABC* is an isosceles triangle with *BA* = *BC* and *AC* = 10 cm.

Find (a) $A\hat{B}C$ in radians,

(b) the area of the shaded region (give your answers correct to 3 significant figures). [6]



27. In the diagram, *PSR* is an arc of a circle, centre *O*, radius *OP* and $P\hat{O}R = \theta$ radians. *OP* = *OR* = *PR* = 12 cm. *PQR* is a semi-circle with *PR* as diameter.

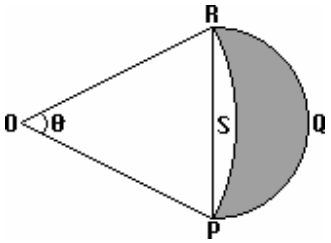
(a) State the value of θ.

(b) Calculate the area of

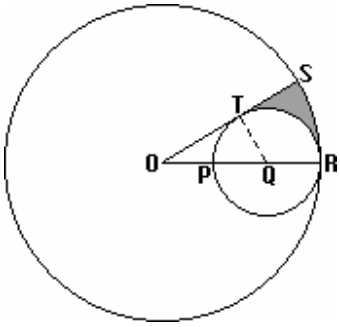
(i) the sector *OPSR* (ii) the segment *PSR*

(iii) the shaded region.

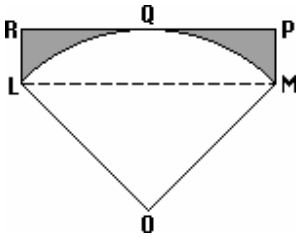
Give your answers to 3 significant figures. [8]



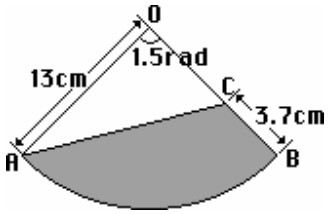
28. In the diagram, the circle centre *Q* touches the circle centre *O* internally at *R*. *OS* is the tangent to the smaller circle at *T*. If the radius of the smaller circle is 4 cm and *OP* = *PR*, find
(a) angle *ROS* and angle *RQT* in radians,
(b) the length of the minor arc *RS*,
(c) the area of the shaded region. [7]



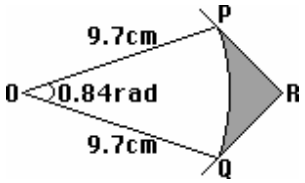
29. In the diagram, *OLQM* is a sector of a circle centre *O*. The segment *LQM* is enclosed in a rectangle *LMPR*. Given that *LM* = 36 cm and *MP* = 12 cm, calculate
(a) the length of *OL*,
(b) the angle *LOM* in radians,
(c) the area of the shaded region. [6]



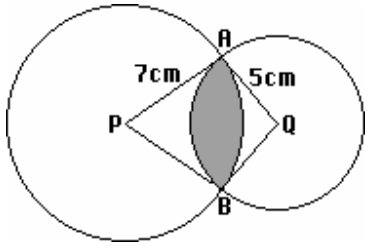
30. In the diagram, *OAB* is a sector of a circle, centre *O* and radius 13 cm. *C* lies on *OB* such that *BC* = 3.7 cm. Given that angle *AOB* = 1.5 rad, find
(a) the length of arc *AB*,
(b) the area of the shaded region. [5]



31. The diagram shows part of a circle, centre *O*, of radius 9.7 cm. The tangents at the points *P* and *Q* on the circumference of the circle meet at the point *R* and the angle *POB* is 0.84 radians. Calculate
(a) the length of the perimeter of the shaded region,
(b) the area of the shaded region. [7]

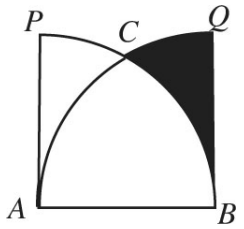


32. The diagram shows a pair of intersecting circles with centres at *P* and *Q* and of radii 7 cm and 5 cm. *AB* is the common chord and is of length 8 cm. Find
(a) angle *APB* in radians,
(b) angle *AQB* in radians,
(c) the shaded area. [7]

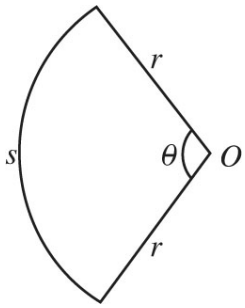


33. The area of a sector is 44 cm² and its perimeter is 30 cm. Find all the possible radii of the circle. [4]

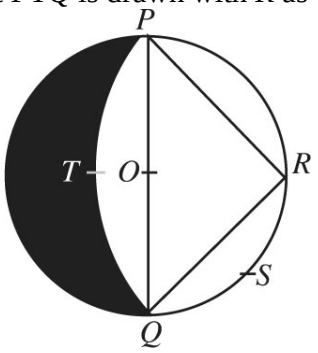
34. *ABP* and *ABQ* are two identical quadrants of a circle. Given that *AB* = 8 cm, find, correct to 4 significant figures,
(a) the perimeter of the shaded region,
(b) the area of the shaded region. [6]



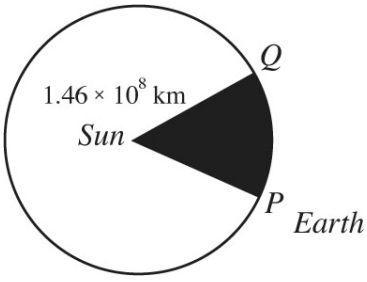
35. In the figure, a wire 4 metres long is bent into a sector of radius *r* and arc length *s*.
(a) Express θ in terms of *r*. [3]
(b) Find the value of *r* such that the area of the sector is maximum. [3]
(c) Find the maximum area and the corresponding central angle. [2]



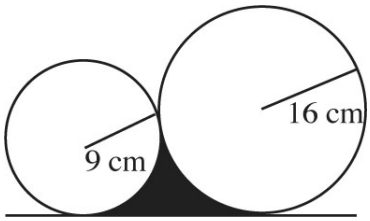
36. *PQ* is a diameter of a circle with center *O* and radius *r* cm. *R* is a point on the circumference of the circle such that *PR* = *QR*. The arc *PTQ* is drawn with *R* as the centre.
(a) Find, in terms of *r*, the area of triangle *PQR* and the area of the shaded region. What can you say about the two areas? Are they equal? [8]
(b) Find the value of the ratio $\frac{\text{area of segment } PTQ}{\text{area of segment } QSR}$. [4]



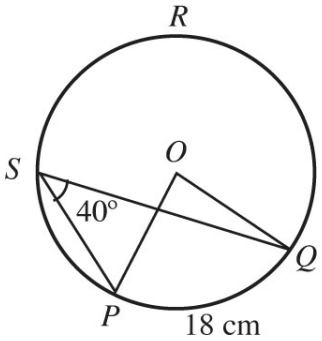
37. Assume that the earth revolves around the sun in a circular orbit with uniform speed of 1.08×10^5 km/hour and that the distance between the sun and the earth is 1.46×10^8 km. The diagram shows that the earth moves from *P* to *Q* in 5 days. Find
(a) the angle subtended at the centre, giving your answer in radians correct to 1 decimal place. [2]
(b) the area of the shaded sector. [2]



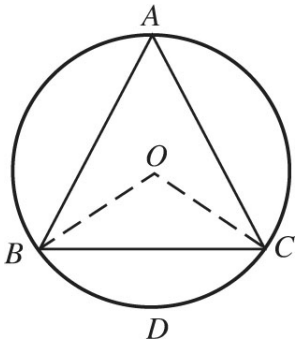
38. The figure shows the cross-sections of two cylindrical logs resting against each other on a level ground. Given that the radii of the cross-sections are 16 cm and 9 cm, find, the perimeter and the area of the shaded region, correct to two decimal places. [6]



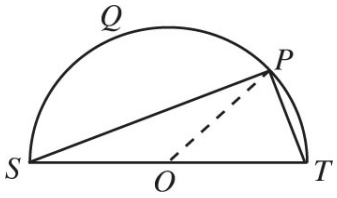
39. In the diagram, O is the centre of the circle. Given that angle $PSQ = 40°$ and the length of arc $PQ = 18$ cm, find
(a) the radius of the circle, [3]
(b) the area of the sector $OQRSP$. [3]



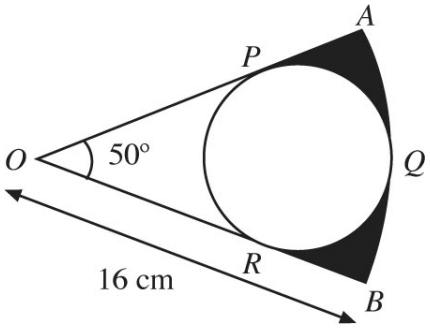
40. The figure shows a circle with centre O and radius 5 cm. A, B and C are three points on the circumference of the circle such that triangle ABC is isosceles and $BC = 8$ cm. Find
(a) the area of the segment BDC, [4]
(b) the value of the ratio $\dfrac{\text{length of arc } AB}{\text{length of chord } AB}$. [3]



41. The figure shows a semicircle with centre O. ST is a diameter and the point P is on the circumference such that the chord $SP = 12$ cm and the chord $TP = 5$ cm.
Calculate, correct to 3 significant figures,
(a) the values, of the angles PST and POT, in radians. [4]
(b) the area of the segment PQS. [3]



42. In the figure, the radius of the sector AOB is 16 cm and angle $AOB = 50^{o}$.
Find, correct to 3 significant figures,
(a) the radius of the inscribed circle PQR, [3]
(b) the area of the shaded region. [9]



Answers

1. (a) (i) 39.2 m (ii) 35.2 m^2 (b) (i) 36.2 cm (ii) 37.3 cm^2

2. (a) 112.8 cm (b) 142.8 cm^2

3. (b) 13 cm (c) 69.372 cm^2

4. (a) 16.7 m (b) 766.3 m^2

5. 4.3 $units^2$

6. (a) 12.6 m (b) 11.0 m (c) 28.6 m (d) 38.9 m^2

7. (a) 10.08 cm (b) 9.45 cm^2

8. (a) 1.68 rad (b) 5.1 cm (c) 16.8 cm^2

9. (a) 10.5 cm (b) 9.1 cm^2 (c) 30.2 cm^2

10. (a) 0.56 rad (b) 24.9 cm (c) 35.7 cm^2

11. (a) 1.25 radians (b) 66.7 cm^2

12. (a) 4 cm (b) 14.2 cm^2

13. (a) 36º
    (b) 44cm

14. (a) 21cm
    (b) 220º

15. (a) 12cm
    (b) 76.4º

16. 79.6cm

17. 209.4cm^2

18. 26.2cm

19. 67cm^2

20. 5.8cm^2

21. 138.2 m^2

22. (a) 1 142 cm^2 (b) 209.45 cm^2 (c) 114.29 cm^2 (d) 32 cm^2

23. 1:2

24. 1.76 cm^2

25. 47.64 cm^2

26. (a) 0.395 rad (b) 60.0 cm^2

27. (a) 1.05 rad (b) (i) 75.4 cm^2 (ii) 13.0 cm^2 (iii) 43.5 cm^2

28. (a) $R\hat{O}S$ = 0.34 rad, $R\hat{Q}T$ = 1.91 rad (b) 5.44 cm (c) 5.59 cm^2

29. (a) 19.5 cm (b) 2.352 rad (c) 119.83 cm^2

30. (a) 19.5 cm (b) 66.45 cm^2

31. (a) 16.8 cm (b) 2.5 cm^2

32. (a) 1.22 rad (b) 1.85 rad (c) 18.0 cm^2

33. 4cm or 11 cm

34. (a) 20.57 cm (b) 10.96 cm^2

35. (a) $\frac{4}{r} - 2$ (b) 1 m (c) 1 m^2, 2 rad

36. (a) r^2, equal area (b) 2

37. (a) 0.09 rad (b) 9.59×10^{14} km^2

38. 61.29 cm, 60.14 cm^2

39. (a) 12.89 cm (b) 406.6 cm^2

40. (a) 3.125 cm^2 (b) 1.238

41. (a) 0.395 rad, 1.316 rad (b) 18.12 cm^2

42. (a) 4.75 cm (b) 17.9 cm^2

Chapter 13

Secondary 3 Mathematics
Chapter 13 Geometrical Properties of Circles

ANSWERS FOR ENRICHMENT ACTIVITIES

Exploration (pg 382)

4 cameras

(a) 6 (b) 3 (c) 2 (d) 2

Secondary 3 Mathematics
Chapter 13 Geometrical Properties of Circles

GENERAL NOTES

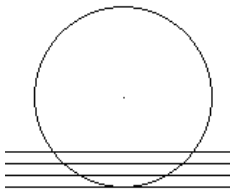
When the diagrams given are straight forward, like those in Qn.1 of Ex 13b, most pupils have no difficulty in recognising the properties of circles such as 'angle at centre is equal to twice the angle at circumference' and 'angles in the same segment are equal'. However, they will find difficulty when the diagrams given are not obvious, where construction lines need to be drawn and when knowledge of angle properties of triangles are required. Where diagrams are not drawn in the obvious manner, the teacher may ask pupils to turn the diagrams around so that they appear more apparent from another angle.

Teachers may find it useful to have transparencies with angles cut out so that it can be moved about to show equal angles.

The teacher may find the Geometer's Sketchpad (GSP) a useful tool to get pupils to verify the results that

(a) the angle at the centre is equal to twice the angle at the circumference subtended by the same arc,
(b) the angle subtended by the diameter of a semicircle is a right angle,
(c) angles in the same segment of a circle are equal, and
(d) the opposite angles of a cyclic quadrilateral are supplementary.

To illustrate the tangent to a circle, use a thin stick (a satay stick will be fine) as a secant, on a transparency, and move it away from the centre of the circle.



To illustrate the properties of tangents from an external point, the method of using reflection may be more easily understood by the pupils.

Referring to Figure 13.18, reflect ΔOAP with OP as the line of reflection to ΔOBP. Since reflection preserves shape and size, ΔOAP and ΔOBP are congruent. $\therefore AP = BP$, $A\hat{P}O = B\hat{P}O$ and $A\hat{O}P = B\hat{O}P$.

XYZ SECONDARY SCHOOL

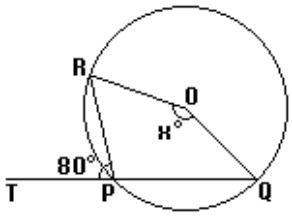
Name: _____ ( ) Date: _____

Time allowed: 35 min

Class: _____ Marks: 16

Secondary 3 Multiple-Choice Questions
Chapter 13 Geometrical Properties of Circles

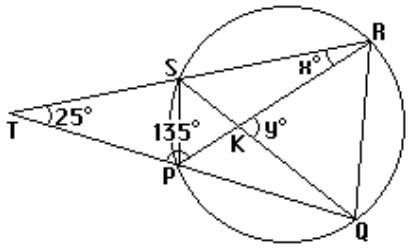
1. In the figure, O is the centre of the circle, $T\hat{P}R = 80°$ and $Q\hat{O}R = x°$. Find x.



(A) 80° (B) 160° (C) 100°
(D) 120° (E) 200°

( )

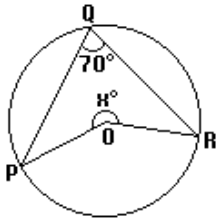
2. In the figure, $S\hat{T}P = 25°$, $T\hat{P}R = 135°$, $S\hat{R}P = x°$ and $R\hat{K}Q = y°$. Calculate the value of $x + y$.



(A) 75° (B) 55° (C) 85°
(D) 70° (E) 80°

( )

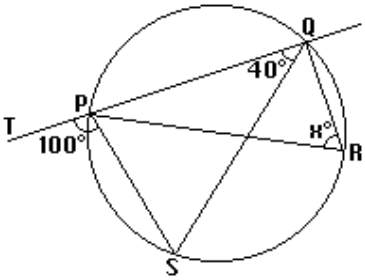
3. In the figure, O is the centre of the circle and $P\hat{Q}R = 70°$. Calculate the value of x.



(A) 140° (B) 210° (C) 290°
(D) 110° (E) 220°

( )

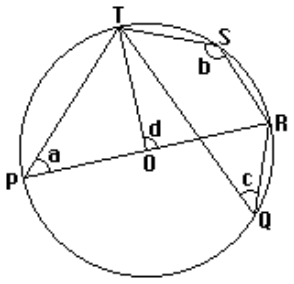
4. In the figure, $S\hat{P}T = 100°$, $P\hat{Q}S = 40°$ and $P\hat{R}Q = x°$. Calculate the value of x.



(A) 30° (B) 40° (C) 50°
(D) 60° (E) 80°

( )

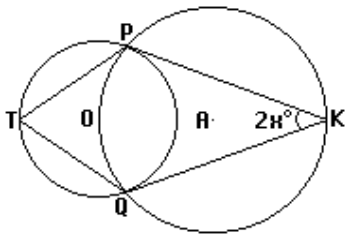
5. In the figure, O is the centre of the figure and PR is a diameter. Which of the following is/are true?



I. $b + d = 180°$ II. $d = 2c$ III. $a = c$

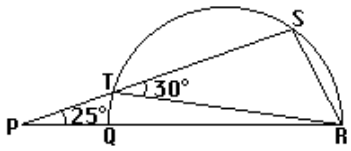
(A) I and II only (B) II and III only
(C) I and III only (D) II only
(E) all of them ( )

6. In the figure, O is the centre of the small circle PTQ and it lies on the circumference of the big circle PKQ whose centre is A. Given that $P\hat{K}Q = 2x°$, express PTQ in terms of x.



(A) $180° - 2x$ (B) $90° - x$
(C) $180° - x$ (D) $90° + x$
(E) $180° - 4x$ ( )

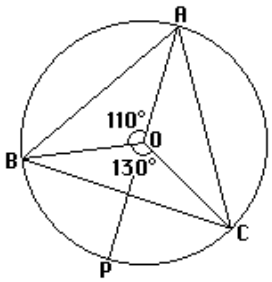
7. In the figure, QR is the diameter of the semi-circle, $S\hat{P}R = 25°$ and $S\hat{T}R = 30°$. Calculate the size of $T\hat{S}R$.



(A) 85° (B) 90° (C) 95°
(D) 125° (E) 105°

( )

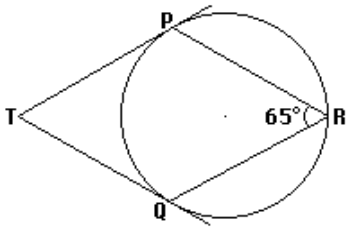
8. In the diagram, O is the centre of the circumscribed circle of ΔABC. AO is produced to meet the circle at P. Given that $A\hat{O}B = 110°$ and $B\hat{O}C = 130°$, find $A\hat{P}C$.



(A) 55° (B) 60° (C) 65°
(D) 120° (E) none of the above

( )

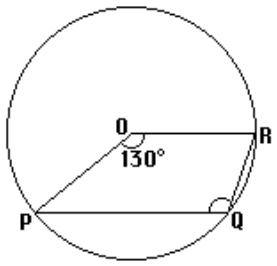
9. In the figure, TP and TQ are tangents to the circle. Given that $P\hat{R}Q = 65°$, calculate $P\hat{T}Q$.



(A) 50° (B) 65° (C) 115°
(D) 130° (E) none of the above

( )

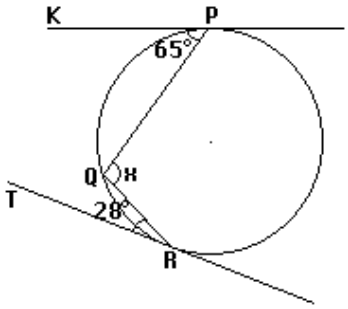
10. In the figure, O is the centre of the circle and $P\hat{O}R = 130°$. Calculate $P\hat{Q}R$.



(A) 60° (B) 65° (C) 115°
(D) 130° (E) none of the above

( )

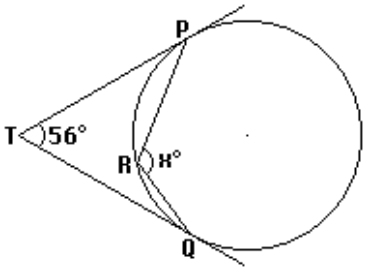
11. In the figure, KP and TR are tangents to t circle, $K\hat{P}Q = 65°$, $T\hat{R}Q = 28°$ and $P\hat{Q}R =$ Find x.



(A) 37° (B) 90° (C) 93°
(D) 87° (E) none of the above

( )

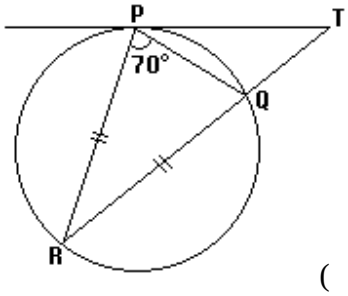
12. In the figure, TP and TQ are tangents to the circle, $P\hat{T}Q = 56°$ and $P\hat{R}Q = x°$. Calculate x.



(A) 112° (B) 118° (C) 124°
(D) 136° (E) cannot be found

( )

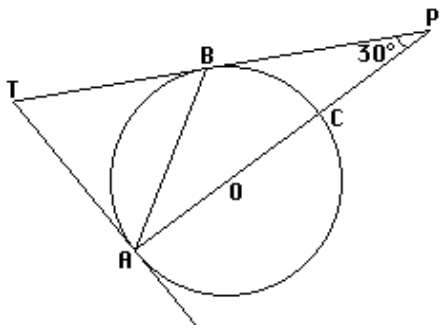
13. In the diagram, PT is the tangent to the circle. Given that $PR = QR$ and $R\hat{P}Q = 70°$, calculate $P\hat{T}Q$.



(A) 30° (B) 40° (C) 50°
(D) 60° (E) 70°

( )

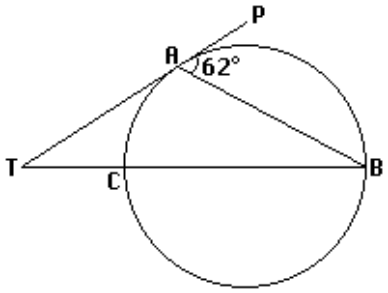
14. In the diagram, TA and TB are tangents to the circle whose centre is O. If $T\hat{P}A = 30°$, find the value of $B\hat{A}C$.



(A) 20° (B) 25° (C) 30°
(D) 35° (E) 60°

( )

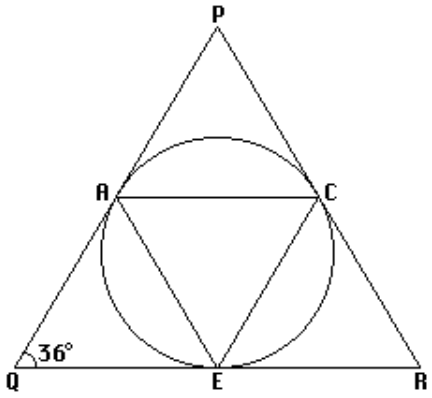
15. In the diagram, BC is the diameter and TA is a tangent to the circle. If $P\hat{A}B = 62°$, calculate A



(A) 28° (B) 30° (C) 32°
(D) 34° (E) 38°

( )

16. In the diagram, PQ, PR and QR are tangents to the circle. If $PQ = PR$ and $P\hat{Q}R = 36°$, calculate $A\hat{P}C$.



(A) 28° (B) 36° (C) 48°
(D) 64° (E) 72°

( )

Answers

1. B	2. C	3. E	4. D
5. B	6. B	7. C	8. B
9. A	10.C	11.D	12.B
13.A	14.C	15.D	16.B

XYZ SECONDARY SCHOOL

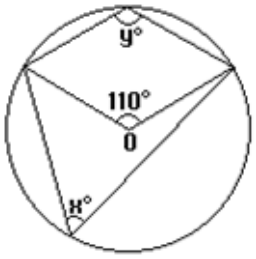
Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks:

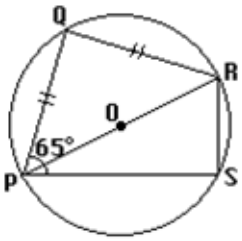
Secondary 3 Mathematics Test
Chapter 13 Geometrical Properties of Circles

1. In the diagram, *O* is the centre of the circle. Calculate the values of



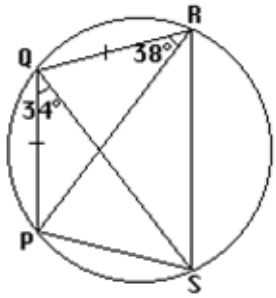
(a) *x*, [1]
(b) *y*. [1]

2. *PR* is the diameter of a circle, centre *O*. The points *Q* and *S* lie on the circumference of the circle such that *PQ* = *RQ* and $Q\hat{P}S$ = 65°.



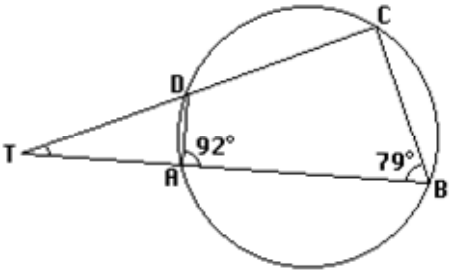
(a) Write down the value of $P\hat{S}R$. [1]
(b) Calculate
(i) $Q\hat{P}R$, [1]
(ii) $P\hat{R}S$. [1]

3. In the diagram, *QP* = *QR*, $P\hat{Q}S$ = 34° and $P\hat{R}Q$ = 38°. Calculate



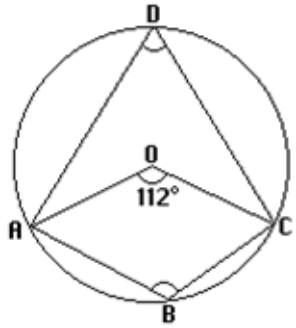
(a) $R\hat{P}S$, [2]
(b) $Q\hat{S}R$. [1]

4. In the diagram, $B\hat{A}D$ = 92° and $A\hat{B}C$ = 79°. Find



(a) $A\hat{T}D$, [2]
(b) $T\hat{C}B$. [1]

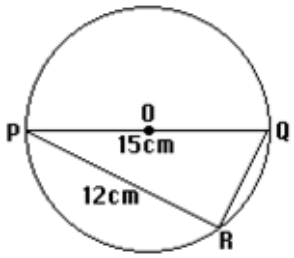
5. In the diagram, O is the centre of the circle. Given that $A\hat{O}C = 112°$, calculate the following.



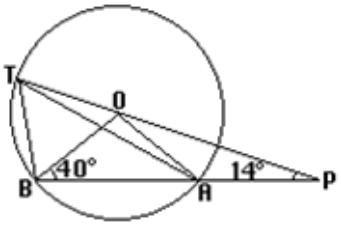
(a) $A\hat{D}C$, [1]

(b) $A\hat{B}C$. [2]

6. PQ is a diameter of the circle, centre O. Given that $PQ = 15$ cm and $PR = 12$ cm, calculate the area of triangle PQR. [3]



7. In the diagram, O is the centre of the circle. PAB and POT are straight lines. Given that $A\hat{P}O = 14°$ and $O\hat{B}A = 40°$, calculate

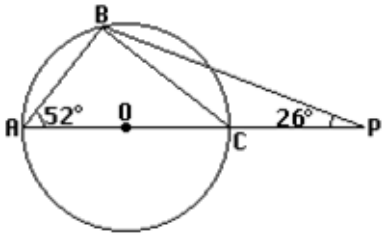


(a) $A\hat{T}B$,

(b) $A\hat{O}P$,

(c) $T\hat{A}B$. [4]

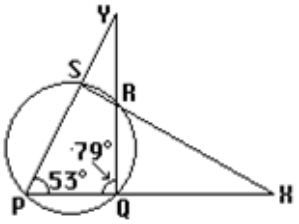
8. In the figure, AC is the diameter of the circle. Given that $B\hat{A}C = 52°$ and $A\hat{P}B = 26°$, calculate



(a) $P\hat{B}C$,

(b) $B\hat{O}A$. [3]

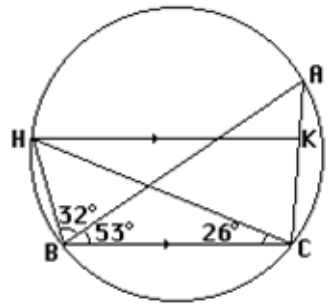
9. In the figure, $S\hat{P}Q = 53°$ and $P\hat{Q}R = 79°$. Find the angles of $S\hat{Y}R$ and $Q\hat{X}R$ [4]



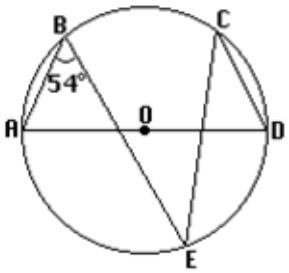
10. In the figure, HK is parallel to BC, $H\hat{B}A = 32°$, $A\hat{B}C = 53°$ and $B\hat{C}H = 26°$. Calculate

(a) $H\hat{K}C$,

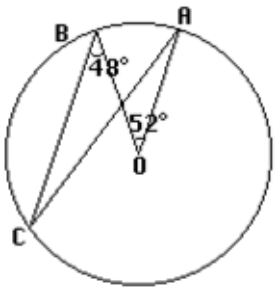
(b) $B\hat{A}C$. [3]



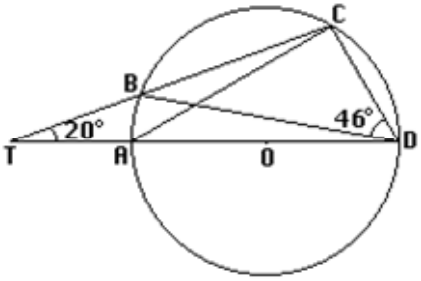
11. In the diagram, O is the centre of the circle and $A\hat{B}E = 54°$. Find the value of $D\hat{C}E$. [3]



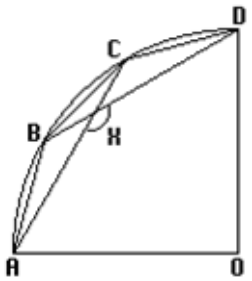
12. In the diagram, O is the centre of the circle, $A\hat{O}B = 52°$ and $O\hat{B}C = 48°$. Calculate the value of $O\hat{A}C$. [3]



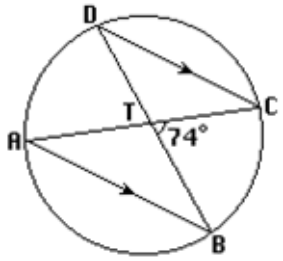
13. In the diagram, $A\hat{O}D$ is a diameter of the circle, $TAOD$ and TBC are straight lines. Given that $A\hat{T}C = 20°$ and $B\hat{D}C = 46°$, find the value of $C\hat{A}D$. [3]



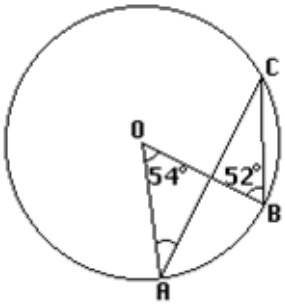
14. In the diagram, *OABCD* is a quadrant of a circle, centre *O*. *AC* and *BD* intersect at the point *X* and *AB* = *BC* = *CD*. Find the value of $A\hat{X}D$. [3]



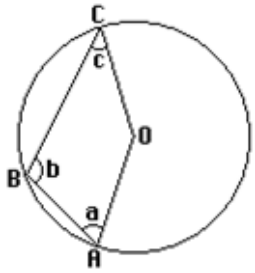
15. In the diagram, *AB* is parallel to *DC*, chords *AC* and *BD* meet at the point *T*. Given that $B\hat{T}C$ = 74°, find $A\hat{C}D$. [3]



16. In the diagram, *O* is the centre of the circle. If $A\hat{O}B$ = 54° and $O\hat{B}C$ = 52°, find the value of $O\hat{A}C$. [3]

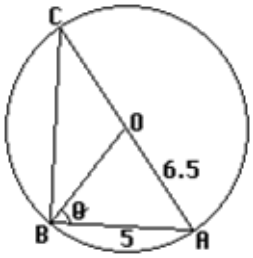


17. In the diagram, *O* is the centre of the circle, $O\hat{A}B = a$, $A\hat{B}C = b$ and $O\hat{C}B = c$. Express *b* in terms of *a* and *c*. [4]

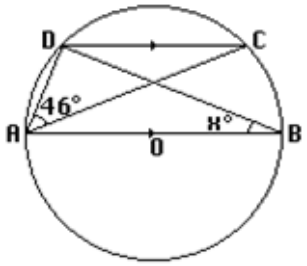


18. In the diagram, O is the centre of the circle of radius 6.5 cm. If AB = 5 cm and $O\hat{B}A = \theta$, find the numerical values of

(a) sin θ, [3]
(b) tan θ. [1]

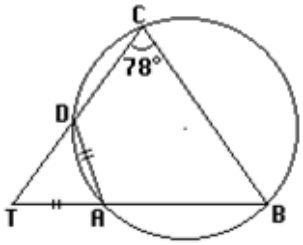


19. In the diagram, AOB is a diameter of the circle, centre O. Given that $AB \parallel DC$, $D\hat{A}C = 46°$ and $A\hat{B}D = x°$, find x. [3]



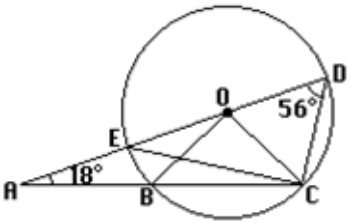
20. In the diagram, TAB is a straight line, $TA = AD$ and $B\hat{C}D = 78°$. Find

(a) $A\hat{T}D$, [2]
(b) $A\hat{B}C$. [2]



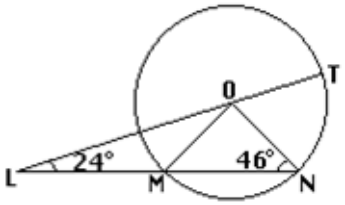
21. In the diagram, O is the centre of the circle. $AEOD$ and ABC are straight lines. If $B\hat{A}O = 18°$ and $A\hat{D}C = 56°$, calculate

(a) $C\hat{E}D$, [1]
(b) $A\hat{C}D$, [1]
(c) $B\hat{O}C$. [2]

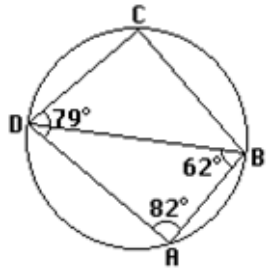


22. In the diagram, O is the centre of the circle. LOT and LMN are straight lines. Given that $O\hat{N}M = 46°$ and $O\hat{L}M = 24°$, calculate

(a) $L\hat{O}M$, [1]
(b) $T\hat{M}N$, [2]
(c) $M\hat{T}N$. [2]



23. In the diagram, *ABCD* is a cyclic quadrilateral in which $A\hat{D}C$ = 79°, $B\hat{A}D$ = 82° and $A\hat{B}D$ = 62°.Find

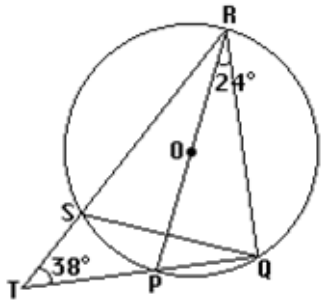


(a) $B\hat{D}C$,
(b) $B\hat{C}D$,
(c) $B\hat{C}A$.

[5]

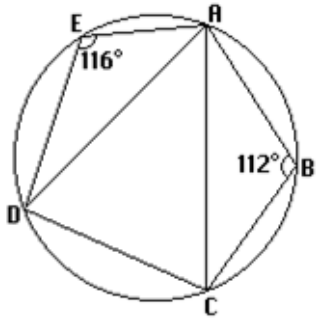
24. *PQ* and *RS* are two parallel chords in a circle of radius 7.5 cm. If *PQ* = 12 cm and *RS* = 9.8 cm, calculate the possible distance between the chords *PQ* and *RS*. [5]

25. In the diagram, *PR* is a diameter of the circle, centre *O*, $P\hat{R}Q$ = 24° and $P\hat{T}R$ = 38°. Calculate



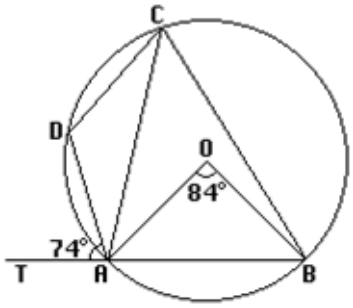
(a) $S\hat{R}P$, [2]
(b) $Q\hat{S}R$, [2]
(c) $P\hat{O}Q$. [2]

26. In the diagram, $A\hat{B}C$ = 112° and $A\hat{E}D$ = 116°. Find the values of



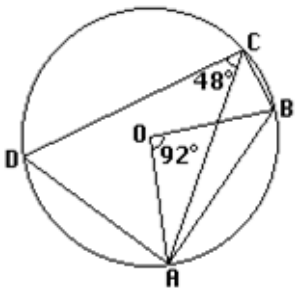
(a) $A\hat{C}D$, [2]
(b) $C\hat{A}D$. [2]

27. In the diagram, *O* is the centre of the circle, *TAB* is a straight line, $A\hat{D}B$ = 84° and $T\hat{A}D$ = 74°. Find the value of



(a) $A\hat{C}B$, [1]
(b) $A\hat{C}D$, [2]
(c) $O\hat{A}D$. [2]

28. In the diagram, O is the centre of the circle, $A\hat{O}B = 92°$ and $A\hat{C}D = 48°$. Find

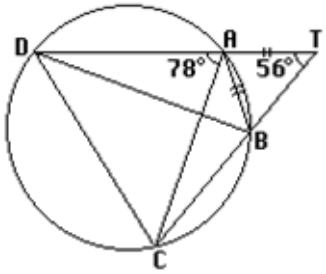


(a) $A\hat{C}B$, [1]

(b) $D\hat{A}B$, [2]

(c) $D\hat{A}O$. [2]

29. In the diagram, TAD and TBC are straight lines where A, B, C and D are points on the circle. Given that $AB = AT$, $D\hat{A}C = 78°$ and $A\hat{T}B = 56°$, calculate

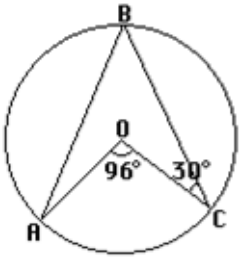


(a) $B\hat{A}C$, [2]

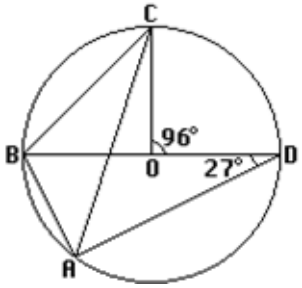
(b) $A\hat{D}B$, [2]

(c) $D\hat{B}C$. [1]

30. In the diagram, O is the centre of the circle, $A\hat{O}C = 96°$ and $O\hat{C}B = 30°$. Calculate the value of $O\hat{A}B$. [3]



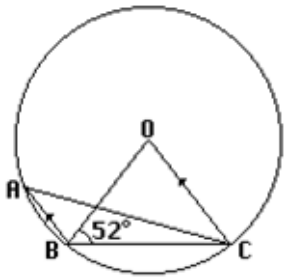
31. In the diagram, O is the centre of the circle, $A\hat{D}B = 27°$ and $C\hat{O}D = 96°$. Calculate



(a) $B\hat{A}C$, [2]

(b) $A\hat{B}C$. [2]

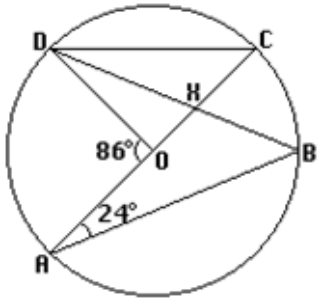
32. In the diagram, *AB* is parallel to *OC*, where *O* is the centre of the circle. Given that $O\hat{B}C = 52°$, calculate



(a) $B\hat{A}C$, [3]

(b) $B\hat{C}A$. [2]

33. In the diagram, *O* is the centre of the circle, *AC* is a diameter and it meets *BD* at *X*. Given that $A\hat{O}D = 86°$ and $B\hat{A}C = 24°$, calculate

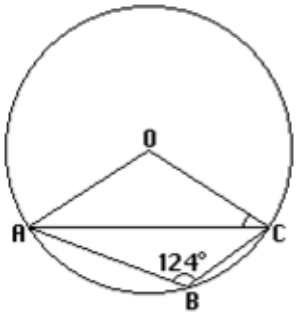


(a) $A\hat{B}X$, [1]

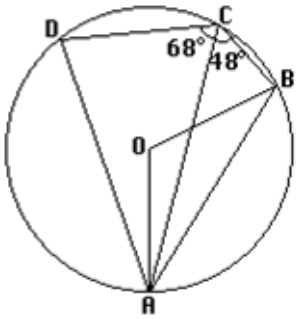
(b) $O\hat{D}X$, [2]

(c) $B\hat{X}C$. [2]

34. In the diagram, *O* is the centre of the circle and $A\hat{B}C = 124°$. Find the value of $O\hat{C}A$. [3]



35. In the diagram, *O* is the centre of the circle. Given that $A\hat{C}B = 48°$ and $A\hat{C}D = 68°$, find the value of



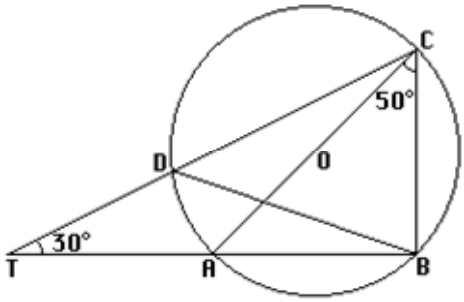
(a) $O\hat{B}A$, [2]

(b) $D\hat{A}O$. [2]

36. In the diagram, *AOC* is the diameter of the circle with centre *O* and *TAB* is a straight line.
Given that $B\hat{T}C$ = 30° and $A\hat{C}B$ = 50°, find

(a) $T\hat{C}A$, [3]

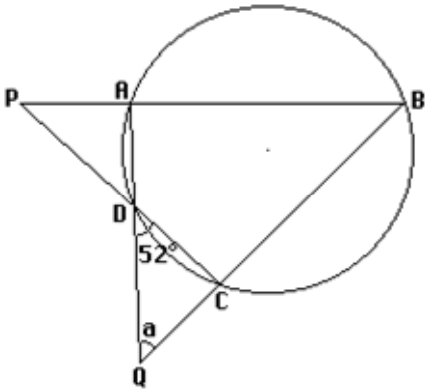
(b) $A\hat{B}D$. [2]



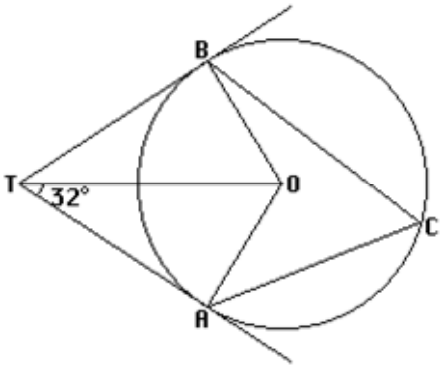
37. In the diagram, *PAB*, *ADQ* and *BCQ* are straight lines. Given that $C\hat{D}Q$ = 52° and $C\hat{Q}D$ = *a*°, find

(a) $B\hat{C}P$ in terms of *a*, [2]

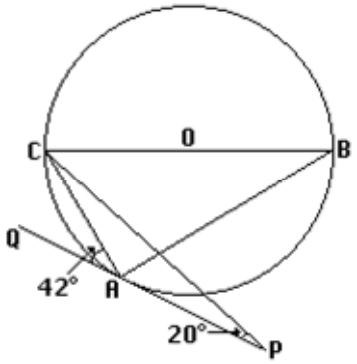
(b) $B\hat{A}D$ when $B\hat{P}C$ = (*a* + 26)° [3]



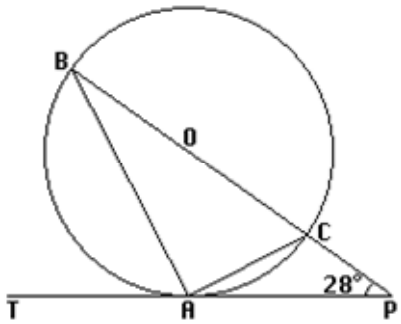
38. In the diagram, *TA* and *TB* are tangents to the circle with centre *O*.
Given that $A\hat{T}O$ = 32°, find $A\hat{C}B$. [3]



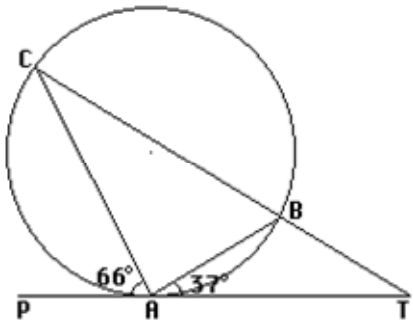
39. In the figure, *BOC* is a diameter of the circle and *PAQ* is the tangent to the circle at *A*.
Given that $A\hat{P}C$ = 20° and $C\hat{A}Q$ = 42°, calculate $B\hat{C}P$. [3]



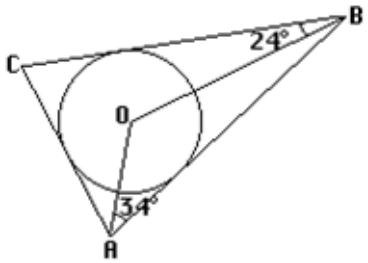
40. In the diagram, *O* is the centre of the circle, *TAP* is the tangent to the circle at the point *A*, *BOCP* is a straight line and $A\hat{P}C$ = 28°. Calculate *TÂB*. [3]



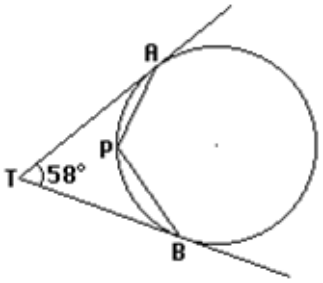
41. In the diagram, *TAP* is the tangent to the circle at *A*. Given that *PÂC* = 66° and *BÂT* = 37°, calculate $A\hat{T}B$. [3]



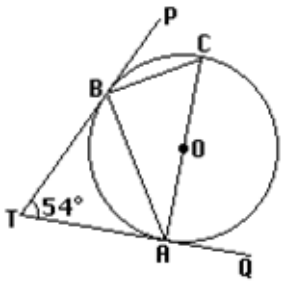
42. In the diagram, *O* is the centre of the circle, *BC*, *AC* and *AB* are tangents to the circle. Given that *OÂB* = 34° and $O\hat{B}C$ = 24°, find the value of $B\hat{C}A$. [3]



43. In the diagram, *TA* and *TB* are tangents to the circle at *A* and *B* respectively. Given that $A\hat{T}B$ = 58°, calculate $A\hat{P}B$. [3]



44. In the diagram, *TA* and *TB* are tangents to the circle whose centre is at *O*. Given that $A\hat{T}B = 54°$, calculate

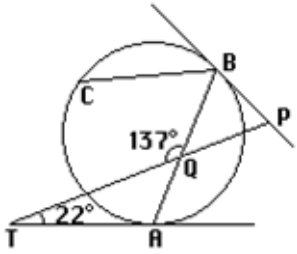


(a) $P\hat{B}C$, [1]

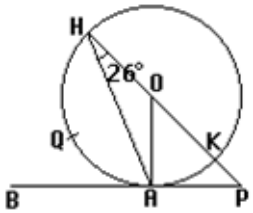
(b) $B\hat{C}A$, [1]

(c) $B\hat{A}Q$. [1]

45. *AQB* is a chord of the circle *ABC*. *TA* is the tangent to the circle at *A*, the tangent to the circle at *B* meets *TQ* produced at *P*. If $A\hat{T}P = 22°$ and $T\hat{Q}B = 137°$, calculate $T\hat{P}B$. [3]



46. The figure shows a circle with centre *O* and radius 8 cm. The diameter *HK* is produced to *P* and *PAB* is a tangent to the circle at *A*. Given that $A\hat{H}O = 26°$, calculate

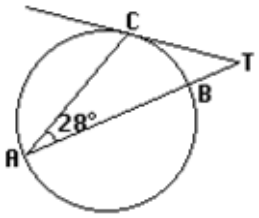


(a) $A\hat{P}K$, [1]

(b) $B\hat{A}H$, [1]

(c) the area of the minor segment *AQH*. [3]
(Take $\pi = 3.14$)

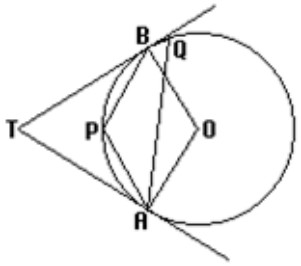
47. In the figure, *AB* is a diameter of the circle and *TC* is a tangent to the circle at *C*. *TC* meets *AB* produced at *T*. If $T\hat{A}C = 28°$, calculate



(a) $A\hat{B}C$,

(b) $A\hat{T}C$. [3]

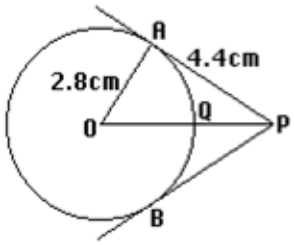
48. In the diagram, *TA* and *TB* are tangents to the circle whose centre is *O*. Given that $A\hat{P}B = 116°$, find



(a) $A\hat{Q}B$, [1]

(b) $A\hat{T}B$. [3]

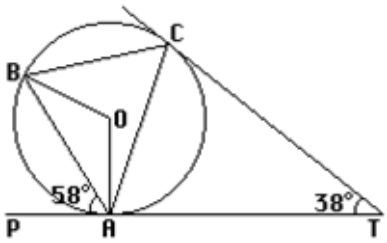
49. In the figure, *PA* and *PB* are tangents to the circle at *A* and *B* respectively. Given that *O* is the centre of the circle, *OA* = 2.8 cm and *AP* = 4.4 cm, calculate



(a) *PQ*,

(b) $A\hat{P}B$. [4]

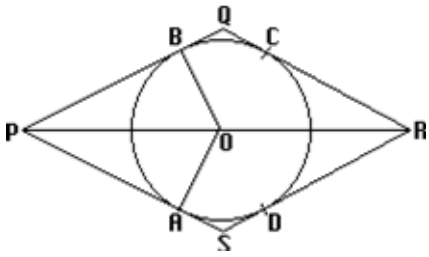
50. In the diagram, *TC* and *TA* are tangents to the circle with centre *O*. Given that $P\hat{A}B = 58°$ and $A\hat{T}C = 38°$, find



(a) $O\hat{B}C$, [2]

(b) $O\hat{A}C$. [2]

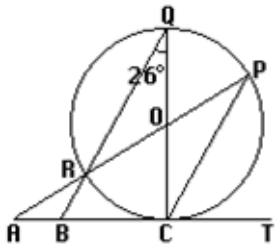
51. In the diagram, *PQ*, *PS*, *RQ* and *RS* are tangents to the circle at *B*, *A*, *C* and *D* respectively. Given that $P\hat{O}A = 56°$ and $S\hat{R}O = 26°$ where *O* is the centre of the circle, calculate



(a) $S\hat{P}O$, [2]

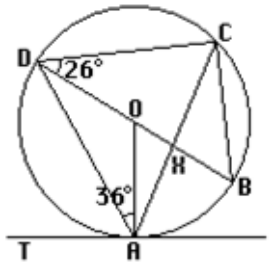
(b) $P\hat{Q}R$. [2]

52. In the diagram, *ACT* is a tangent to the circle at *C*. Given that *COQ* is a diameter, *AROP* is a straight line and $B\hat{Q}C$ = 26°, calculate



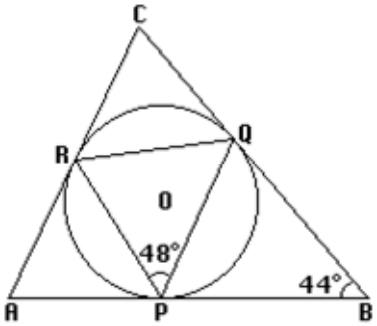
(a) $R\hat{P}C$, [2]
(b) $P\hat{A}T$, [2]
(c) $P\hat{C}T$. [1]

53. In the diagram, *O* is the centre of the circle and *TA* is the tangent to the circle at *A*. *T*he diameter *BD* and *AC* meet at *X*. Given that $B\hat{D}C$ = 26° and $D\hat{A}O$ = 36°, calculate



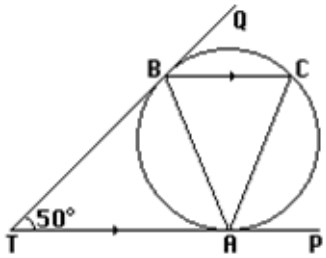
(a) $B\hat{A}T$, [2]
(b) $O\hat{B}A$, [1]
(c) $A\hat{B}C$, [1]
(d) $B\hat{X}C$. [2]

54. In the diagram, *AB*, *BC* and *AC* are tangents to the circle at *P*, *Q* and *R* respectively. If $A\hat{B}C$ = 44° and $Q\hat{P}R$ = 48°, calculate



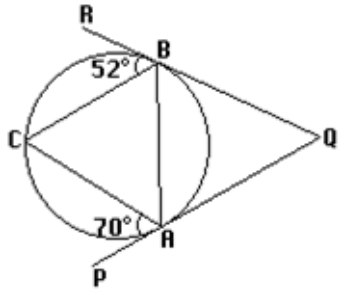
(a) $B\hat{A}C$, [3]
(b) $P\hat{Q}R$. [2]

55. In the diagram, *TAP* and *TBQ* are tangents to the circle at *A* and *B* respectively. Given that *TAP* is parallel to *BC* and $A\hat{T}B$ = 50°, calculate



(a) *PÂC*, [3]
(b) *BÂC*. [2]

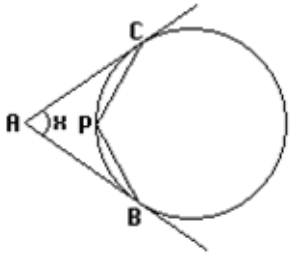
56. In the figure, *PAQ* and *QBR* are tangents to the the circle at *A* and *B* respectively. Given that $P\hat{A}C = 70°$ and $R\hat{B}C = 52°$, calculate



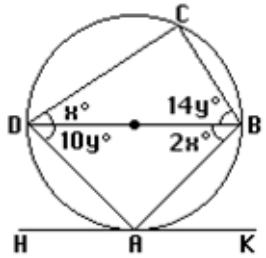
(a) $A\hat{Q}B$, [2]

(b) $A\hat{C}B$. [2]

57. In the diagram, *AB* and *AC* are tangents to the circle at *B* and *C* respectively. Given that $B\hat{A}C = x°$, express $B\hat{P}C$ in terms of *x*. [4]



58. In the diagram, *O* is the centre of the circle and *HK* is a tangent to the circle at *A*. Given that $A\hat{D}B = 10y°$, $A\hat{B}D = 2x°$, $C\hat{D}B = x°$ and $D\hat{B}C = 14y°$, calculate, in degrees,

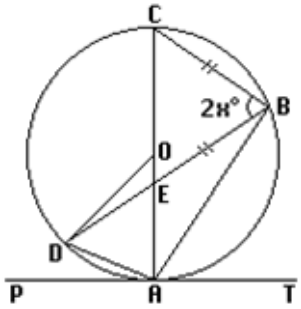


(a) $C\hat{O}B$, [3]

(b) $A\hat{C}D$, [1]

(c) $K\hat{A}B$. [1]

59. In the diagram, *PAT* is a tangent to the circle at *A*, *AC* is a diameter, *BED* is a straight line and *BE* = *BC*. Given that $C\hat{B}E = 2x°$, find in terms of *x*

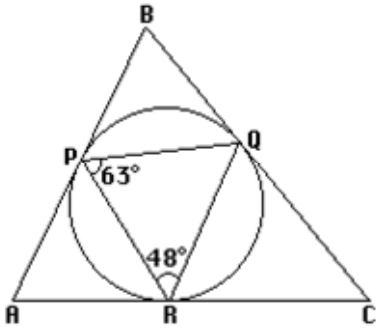


(a) *AÔD*, [2]

(b) *BÂT*, [2]

(c) *PÂD*. [2]

60. In the figure, *PQR* is a triangle inscribed in the circle. *AB*, *BC* and *AC* are tangents to the circle. Given that $Q\hat{P}R$ = 63° and $P\hat{R}Q$ = 48°, calculate

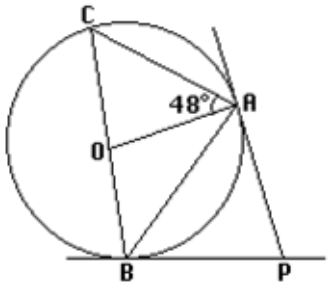


(a) $A\hat{B}C$, [2]

(b) $B\hat{A}C$, [2]

(c) $A\hat{C}B$. [1]

61. *PA* and *PB* are tangents to a circle, centre *O*, touching the circle at *A* and *B*. *BOC* is a diameter and *CÂO* = 48°. Calculate

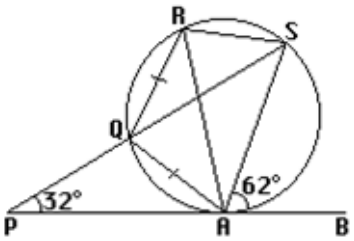


(a) $A\hat{O}C$,

(b) $A\hat{B}O$,

(c) $A\hat{P}B$. [6]

62. In the figure, *PAB* is the tangent to the circle at *A* and *PQS* is a straight line. Given that *AQ* = *QR*, *BÂS* = 62° and $A\hat{P}Q$ = 32°, calculate

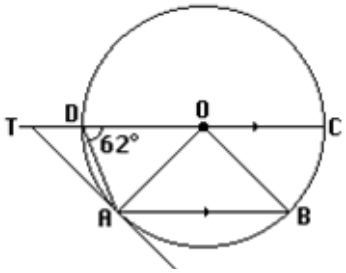


(a) $Q\hat{A}P$,

(b) $Q\hat{R}S$

(c) $R\hat{S}A$. [5]

63. In the diagram, *TA* is a tangent to the circle. *AB* is parallel to the diameter *DOC*. *CODT* is a straight line and $A\hat{D}C$ = 62°.



(a) Give a brief reason why *TÂO* = 90°. [1]

(b) Name a pair of congruent triangles and give a brief reason to substantiate your claim. [2]

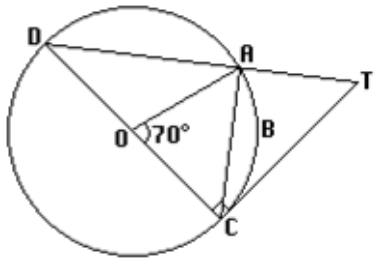
(c) Calculate the value of

(i) $T\hat{A}D$, [1]

(ii) $C\hat{T}A$, [1]

(iii) $B\hat{O}C$. [1]

64. *CT* is a tangent to a circle, centre *O* and radius 6 cm. *DOC* and *DAT* are straight lines and $A\hat{O}C = 70°$. Calculate



(a) the area of the sector *OABC*, taking π to be $\frac{22}{7}$, [2]

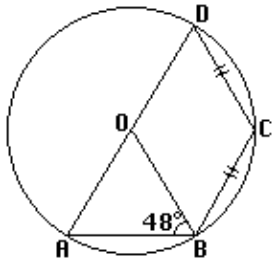
(b) the area of ΔOAC, [2]

(c) the area of the minor segment *ABC*, [2]

(d) $C\hat{D}A$, [2]

(e) the length of the tangent *CT*. [2]

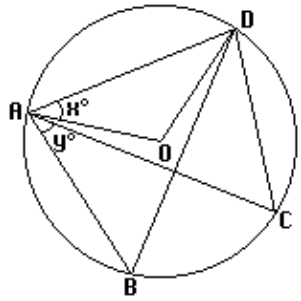
65. In the diagram, *O* is the centre of the circle, *AD* is the diameter, $O\hat{B}A = 48°$ and *BC* = *CD*. Calculate the following:



(a) $O\hat{A}B$ (b) $A\hat{B}C$

(c) $B\hat{C}D$ (d) $A\hat{D}C$ [8]

66. In the figure, *O* is the centre of the circle. The chords *AC* and *BD* are perpendicular, $O\hat{A}D = x°$ and $B\hat{A}C = y°$.

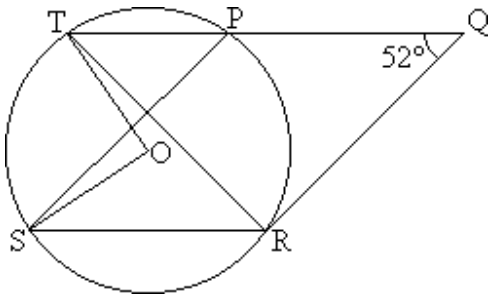


(a) Express $A\hat{O}D$ in terms of *x*.

(b) Express $A\hat{C}D$ in terms of *y*.

(c) Show that *x* = *y*. [6]

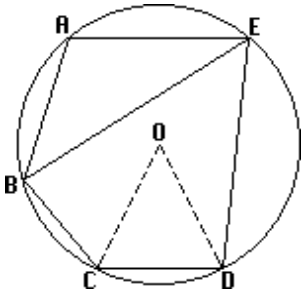
67. In the diagram, *O* is the centre of the circle and *PQRS* is a parallelogram. Given that $P\hat{Q}R = 52°$, calculate the following:



(a) *PTR* (b) $S\hat{O}T$ [4]

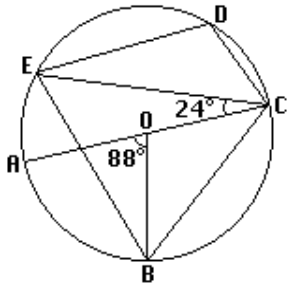
68. The diagram shows a pentagon *ABCDE* inscribed in a circle with centre *O*. Given that $AB = BC = CD$ and $A\hat{B}C = 130°$, calculate the following:

(a) $A\hat{E}B$ (b) $A\hat{E}D$ (c) $C\hat{O}D$ [6]



69. In the diagram, *O* is the centre of the circle, *AOC* is a diameter and *CD* is parallel to *BE*. Given that $A\hat{O}B = 88°$ and $A\hat{C}E = 24°$, calculate the following:

(a) $B\hat{E}C$ (b) $B\hat{C}D$ (c) $C\hat{E}D$ [6]

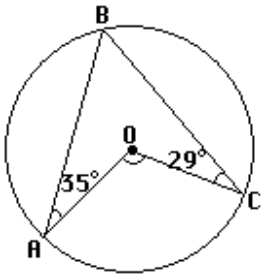


70. An equilateral triangle of side 12 cm is drawn in a circle with its edges touching the circumference. Calculate the radius of the circle. [4]

71. An isosceles triangle of sides 6 cm, 6 cm and 8 cm long is inscribed in a circle. Find the radius of the circle. [4]

72. In a circle of diameter 12 cm, *PQ* and *HK* are two parallel chords of lengths 8 cm and 5 cm respectively. Calculate the distance between the chords if they are on
(a) the same side of the centre, (b) opposite sides of the centre. [4]

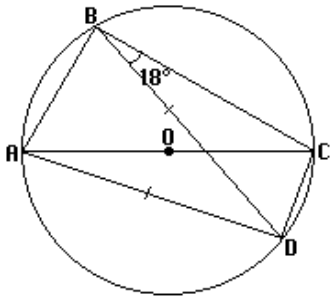
73. In the figure, *O* is the centre of the circle. If $B\hat{A}O = 35°$ and $B\hat{C}O = 29°$, calculate $A\hat{O}C$ as indicated in the diagram. [3]



74. In the figure, AC is the diameter of the circle whose centre is O. If $AD = BD$ and $C\hat{B}D = 18°$, calculate the following angles:

(a) $A\hat{D}B$ (b) $B\hat{C}D$ (c) $C\hat{A}D$

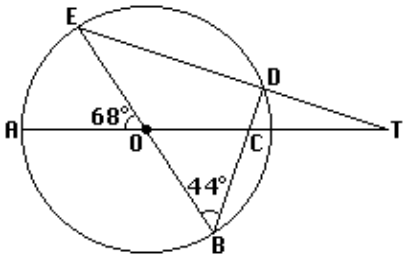
[6]



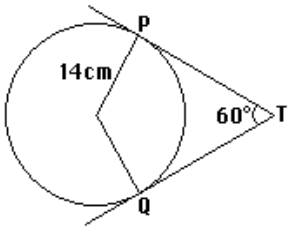
75. In the figure, O is the centre of the circle. $A\hat{O}E = 68°$ and $O\hat{B}D = 44°$. Calculate the following angles:

(a) $B\hat{E}D$ (b) $O\hat{C}B$ (c) $O\hat{T}E$

[6]



76. In the diagram, PT, QT are tangents to the circle and $P\hat{T}Q = 60°$. If the radius of the circle is 14 cm, find the length of the minor arc PQ. (Take $\pi = \frac{22}{7}$) [3]

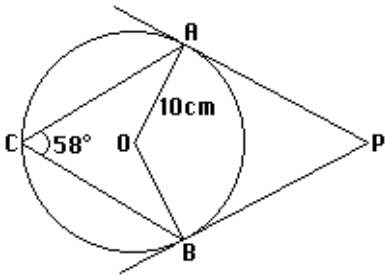


77. In the diagram, PA and PB are tangents to a circle with centre O. If $A\hat{C}B = 58°$, $AO = 10$ cm, find the following.

(a) $A\hat{O}B$ (b) $O\hat{A}B$

(c) $A\hat{P}B$ (d) the length of AP

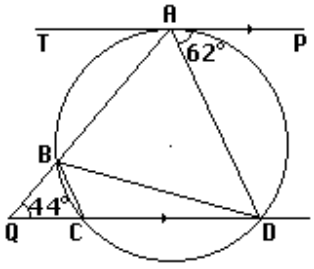
[8]



78. In the diagram, TAP is a tangent to the circle at A, TAP is parallel to QCD, $A\hat{Q}C = 44°$ and $P\hat{A}D = 62°$. Calculate

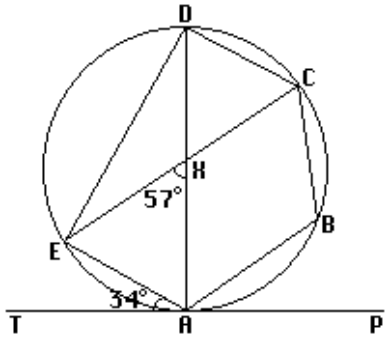
(a) $B\hat{C}D$, (b) $A\hat{D}B$, (c) $B\hat{D}C$.

[6]



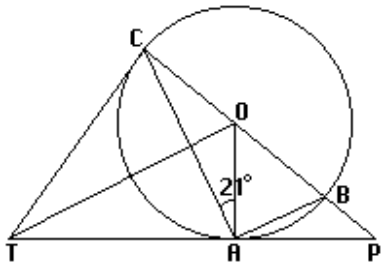
79. In the diagram, *TAP* is a tangent to the circle at *A*. Given that $A\hat{X}E = 57°$ and $T\hat{A}E = 34°$, if *AD* is a diameter, find

(a) $A\hat{E}X$,
(b) $A\hat{B}C$,
(c) $D\hat{E}X$. [6]



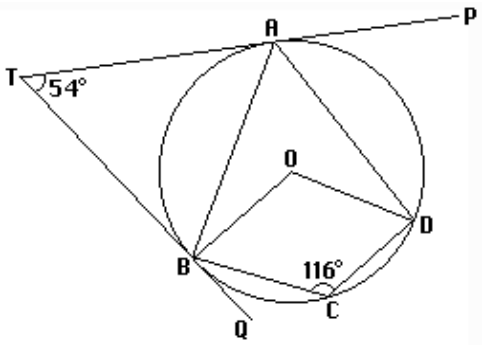
80. In the diagram, *TC* and *TAP* are tangents to the circle with centre *O*. Given that $C\hat{A}O = 21°$, calculate

(a) $A\hat{P}B$, (b) $A\hat{T}C$, (c) $P\hat{A}B$. [6]



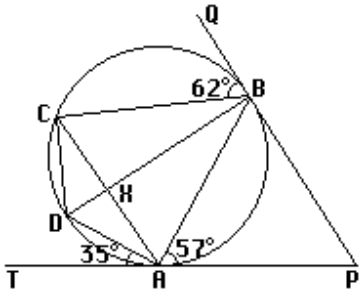
81. In the figure, *O* is the centre of the circle. *TAP* and *TBQ* are tangents to the circle at *A* and *B* respectively. Given that $A\hat{T}B = 54°$ and $B\hat{C}D = 116°$, calculate

(a) $P\hat{A}D$, (b) $A\hat{B}O$. [4]

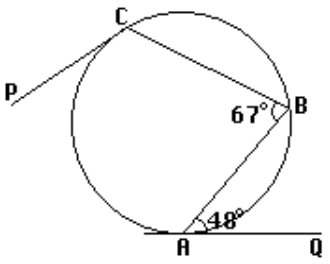


82. In the diagram, *PA* and *PB* are tangents to the circle at *A* and *B*. The chords *AC* and *BD* meet at *X*. Given that $D\hat{A}T = 35°$, $P\hat{A}B = 57°$ and $C\hat{B}Q = 62°$, calculate the following:

(a) $A\hat{P}B$ (b) $A\hat{B}C$ (c) $B\hat{X}C$ [6]

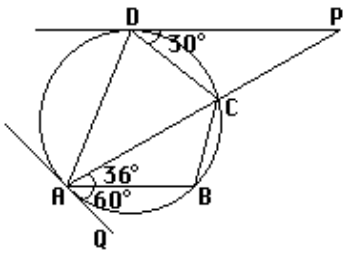


83. In the diagram, AQ and CP are tangents to the circle at A and C respectively. Given that $B\hat{A}Q = 48°$ and $A\hat{B}C = 67°$, calculate $B\hat{C}P$. [3]



84. In the diagram, AQ and DP are tangents to the circle at A and D respectively. Given that $P\hat{D}C = 30°$, $Q\hat{A}B = 60°$ and $B\hat{A}C = 36°$, calculate the following:

(a) $A\hat{B}C$ (b) $A\hat{C}D$ [5]



Answers

1. (a) 55 (b) 125

2. (a) 90° (b) (i) 45° (ii) 70°

3. (a) 70° (b) 38°

4. (a) 13° (b) 88°

5. (a) 56° (b) 124°

6. 54 cm^2

7. (a) 50° (b) 26° (c) 27°

8. (a) 12° (b) 76°

9. 48° ; 26°

10. (a) 122° (b) 69°

11. 36°

12. 54°

13. 32°

14. 150°

15. 37°

16. 25°

17. $b = a + c$

18. (a) $\frac{12}{13}$ (b) $\frac{12}{5}$

19. 22°

20. (a) 51° (b) 51°

21. (a) 34° (b) 106° (c) 80°

22. (a) 22° (b) 35° (c) 44°

23. (a) 43° (b) 98° (c) 36°

24. 1.18 cm or 10.18 cm

25. (a) 28° (b) 66° (c) 48°

26. (a) 64° (b) 48°

27. (a) 42° (b) 32° (c) 58°

28. (a) 46° (b) 86° (c) 42°

29. (a) 34° (b) 22° (c) 78°

30. 18°

31. (a) 42° (b) 111°

32. (a) 38° (b) 14°

33. (a) 43° (b) 19° (c) 67°

34. 34°

35. (a) 42° (b) 22°

36. (a) 10° (b) 10°

37. (a) $(52 + a)°$ (b) 103°

38. 58°

39. 26°

40. 59°

41. 29°

42. 64°

43. 119°

44. (a) 27° (b) 63° (c) 117°

45. 72°

46. (a) 38° (b) 64° (c) 46.2 cm^2

47. (a) 62° (b) 34°

48. (a) 64° (b) 52°

49. (a) 2.42 cm (b) 64.9°

50. (a) 39° (b) 19°

51. (a) 34° (b) 120°

52. (a) 26° (b) 38° (c) 64°

53. (a) 144° (b) 54° (c) 118° (d) 80°

54. (a) 52° (b) 64°

55. (a) 65° (b) 50°

56. (a) 64° (b) 58°

57. $(90 + \frac{x}{2})°$

58. (a) 40° (b) 40° (c) 50°

59. (a) $180° - 4x°$ (b) $90° - x°$ (c) $90° - 2x°$

60. (a) 84° (b) 42° (c) 54°

61. (a) 84° (b) 42° (c) 84°

62. (a) 30° (b) 92° (c) 60°

63. (a) radius is perpendicular to tangent (b) ΔOAD, ΔOBC ; SAS
(c) (i) 28° (ii) 34° (iii) 56°

64. (a) 22 cm^2 (b) 16.9 cm^2 (c) 5.1 cm^2
(d) 35° (e) 8.4 cm^2 0.7

65. (a) 48° (b) 114° (c) 132° (d) 66°

66. (a) $180° - 2x$ (b) $90° - y$

67. (a) 52° (b) 104°

68. (a) 25° (b) 75° (c) 50°

69. (a) 46° (b) 114° (c) 20°

70. 6.93 cm

71. 4.02 cm

72. (a) 0.98 cm (b) 9.93 cm

73. 128 °

74. (a) 36° (b) 108° (c) 18°

75. (a) 46° (b) 62° (c) 22°

76. $29\frac{1}{3}$ cm

77. (a) 116° (b) 32° (c) 64° (d) 16 cm

78. (a) 106° (b) 44° (c) 18°

79. (a) 67° (b) 113° (c) 23°

80. (a) 48° (b) 42° (c) 21°

81. (a) 53° (b) 27°

82. (a) 55° (b) 63° (c) 95°

83. 113°

84. (a) 84° (b) 54°